1. (60 points) Atmospheric CO2 and its relation to anthropogenic emissions (60 points).

Atmospheric carbon dioxide (CO2) is rising due to land use change and fossil fuel combustion. The amount of CO2 due to humans, CO2\text{\text{Human}}, is described by the following equation:

\[
\frac{d(\text{CO}_2^{\text{Human}})}{dt} = \text{Emission} - \frac{\text{CO}_2^{\text{Human}}}{\tau}
\]  

(1)

where Emission is the industrial source of CO2 and \(\tau\) is the lifetime for removal of CO2.

The total atmospheric abundance of CO2 is given by:

\[
\text{CO}_2^{\text{Total}} = \text{CO}_2^{\text{Natural}} + \text{CO}_2^{\text{Human}}
\]  

(2)

where CO2\text{\text{Natural}} represents the natural (pre-industrial) level of atmospheric CO2 (i.e., CO2\text{\text{Human}} = 0 before humans began burning fossil fuels). As we have seen, CO2\text{\text{Natural}} varies on geological time, but here we assume CO2\text{\text{Natural}} = 280 ppm (parts per million). Also, throughout, assume the lifetime for removal of CO2 is 173 yrs.

Equation (1) can be integrated to yield the following solution:

\[
\text{CO}_2^{\text{Human}}(t) = \tau \text{Emission} \left(1 - e^{-t/\tau}\right) + \text{CO}_2^{\text{Human}} \left.t\right|_{\text{Initial}} e^{-t/\tau}
\]  

(3)

a) (10 points) Assuming that only half of the industrial source of CO2 remains airborne; i.e., that the biosphere and ocean will continue to remove half of the emitted CO2, derive an expression for CO2\text{\text{Total}} in the limit of \(t \rightarrow \infty\).

Note: you will have to modify Eq (3) above to account for biospheric and ocean uptake of CO2.

b) (10 points) Suppose that the world’s population, which aspires to live an affluent lifestyle, all of the sudden emits carbon at the same per capita emission as the United States. The present population of the United States is 318 million people and the present world population is 7.3 billion people. Using the value of per capita emission of carbon for the US at the present shown in Lecture 18, calculate how
much carbon would be emitted, in units of Gt C / year, should the world’s population stay fixed at 7.3 billion people and should everyone emit, for perpetuity, at the same per capita emission as the US.

c) (10 points) If the scenario described in part b) were to occur, estimate $\text{CO}_2^{\text{Total}}$ as time approaches $\infty$ using the expression found in a) and the Emission value found in b).

Note: to complete this calculation as well as d), e), & f) you will have to recall the relation between 1 ppm of atmospheric CO$_2$ and 1 Gt of carbon, which was covered in the early part of this class. Also, assume all of biospheric and oceanic uptake continue to absorb half of the emitted CO$_2$.

d) (10 points) Jim Hansen has termed $\text{CO}_2^{\text{Total}}$ of 560 ppm a tipping point that “could lead to a disaster of unimaginable proportions”, were this level of atmospheric CO$_2$ to be reached.

Suppose that the world’s governments decide to reduce global carbon emissions such that the level of atmospheric CO$_2$ (i.e., $\text{CO}_2^{\text{Total}}$) will stabilize at 560 ppm ($2 \times \text{CO}_2^{\text{Natural}}$) in the distant future. What C emission value, in units of Gt C/yr, would be needed to stabilize $\text{CO}_2^{\text{Total}}$ at this level?

e) (10 points) Making a reasonable estimate of how much world population will rise in year 2060, estimate the per capita emission of C, in units of tons of C per person, needed to achieve stabilization of CO$_2$ at 560 ppm. In your reply, describe how you have projected population forward to year 2060 and of course, please be clear to state your estimate of world population in 2060.

f) (10 points) In a paragraph (3 to 4 sentences), discuss whether or not you believe the estimate of C emission found in e) is a reasonable goal for the world to aspire to achieve. If so, state (in broad terms) the means to achieve this goal. If not, state why. There is no right or wrong answer here and the specifics of achieving a particular goal should be saved for Problem Set #6.
2. US Energy Needs & Solar PVs (40 points). According to http://www.indexmundi.com/g/g.aspx?c=us&v=81, the US consumes 3741 billion kWh of electricity in 2012. According to a slide shown in Lecture 19, renewable sources provided about 13% of this demand, with the majority of this sector provided by wind and hydro. Here, we will explore the economics and viability of using traditional solar photovoltaic (PV) panels to meet this entire U.S. electricity need. The following relations may be of use:

\[
1 \text{ kilowatt-hr} = 3.6 \times 10^6 \text{ J} \\
1 \text{ kilowatt} = 10^3 \text{ Watt} \\
1 \text{ W} = 1 \text{ J/s}
\]

There are 80 million single-family homes (1-unit, detached) homes in the United States, according to http://www.census.gov/compendia/statab/2011/tables/11s0984.pdf

Assume that the U.S. government decides to place a 5 kilowatt solar PV panel array on the roof of every single family home in the nation: **5 kilowatt refers to the output of this system at noon (peak sun), for clear sky conditions.**

a) (10 points) What fraction of the US current electricity consumption would be provided if a 5 kilowatt solar PV system was placed on the roof of every US single family home?

Note: in arriving at this estimate, please take into consideration the fact these systems only produce full energy under clear sky conditions, for overhead sun (local noon during summer). Provide a “reasonable estimate” of the annual electricity output from solar PV panel arrays, taking into consideration factors such as day vs night, clear sky vs cloudy sky, and that the sun sweeps through the sky each day (we are looking for rough, “back of the envelope” estimates).

b) (10 points) Assume each system costs $4.70 per watt of output (installed), the present price stated at http://upload.wikimedia.org/wikipedia/commons/7/71/Price_history_of_silicon_PV_cells_since_1977.svg.

Given that the gross domestic product (GDP) of the United Stated is presently estimated to be $17.7 trillion, what fraction of the US GDP would be required to install a 5 kilowatt solar PV system on the rooftop of every single family home in the United States?

c) (20 points) If you were advising the U.S. government, would you recommend the government invest in the installation of a 5 kilowatt solar PV panel array on the roof top of every single family home? Please write a paragraph (2 to 4 sentences) using the numerical estimates found in a) and b) to justify your reply.