

# Fundamentals of Earth's Atmosphere

AOSC 433/633 & CHEM 433

Ross Salawitch

**Class Web Site:** <http://www.atmos.umd.edu/~rjs/class/spr2015>

## Goals:

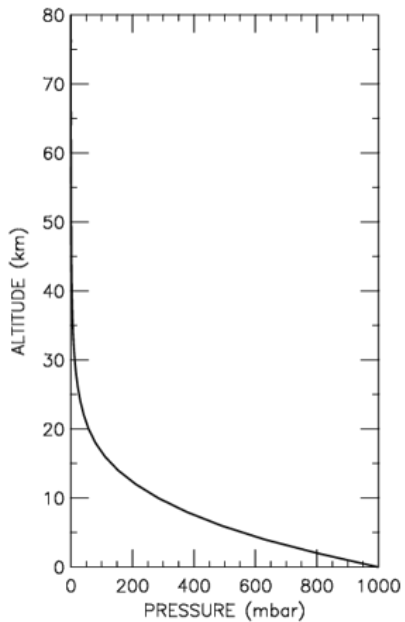
- 1) Tie up loose ends from last lecture
- 2) Barometric law (pressure vs height)
- 3) Thermal structure (temperature vs height)
- 4) Geostrophy (balance of pressure force & Coriolis Force  $\Rightarrow$  storms)
- 5) Ferrel Cell (mean circulation Earth's atmosphere  $\Rightarrow$  climate regimes)

**Lecture 3**

**3 February 2015**



# Pressure versus Altitude



- We can calculate the mass of Earth's atmosphere, using  $p_{\text{STANDARD}}$ , as follows:

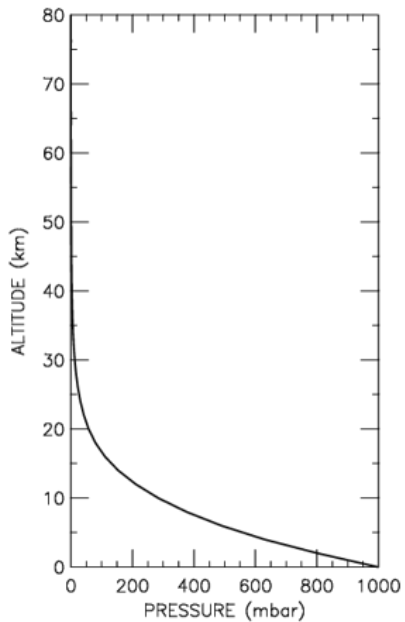
1)  $\text{Pressure} = \text{Force} / \text{Area} = (\text{Mass} \times \text{Acceleration}) / \text{Area}$

2)  $\text{Area of Earth} = 4 \pi R_{\text{EARTH}}^2 = 4 \pi (6371 \text{ km})^2 = 5.1 \times 10^8 \text{ km}^2 = 5.1 \times 10^{18} \text{ cm}^2$

3)  $\text{Accel of Gravity at Earth's surface, } grav, \text{ is } 981 \text{ cm sec}^{-2}$

4)  $\text{Therefore Mass of Earth's Atmosphere} = 1.01325 \times 10^6 \text{ gm} / \text{cm sec}^2$   
 $\times 5.1 \times 10^{18} \text{ cm}^2 / 981 \text{ cm sec}^{-2}$   
 $= 5.27 \times 10^{21} \text{ gm}$

# Pressure versus Altitude



- **Barometric law describes the variation of Earth's pressure with respect to altitude:**

$$\text{Pressure (z)} = \text{Pressure (surface)} \times e^{-z/H}$$

where  $H$  is called the “scale height”

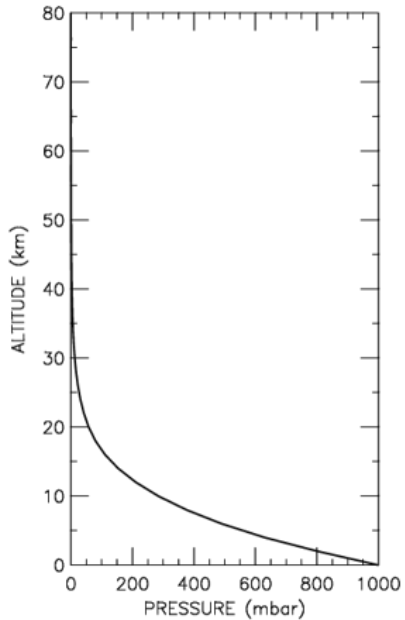
$$\text{Can show } H = R_{\text{EARTH}} T(z) / \text{grav} ,$$

$$\text{where } R_{\text{EARTH}} = R_{\text{UNIVERSAL}} / \text{MW}_{\text{EARTH ATMOS}}$$

$$= 8.3143 \times 10^6 \text{ ergs / K mole} / (28.8 \text{ gm / mole})$$

$$= 2.88 \times 10^6 \text{ ergs / K gm}$$

# Pressure versus Altitude



Derivation of the Barometric Law involves use of the Ideal Gas Law:

$$p \text{ Vol} = n R T$$

where  $p$  is pressure,  $\text{Vol}$  is volume,  $n$  is the number of moles of a gas,

$R$  is the gas constant ( $8.3143 \times 10^7 \frac{\text{ergs}}{\text{K mole}}$ ), and  $T$  is temperature.

- **Barometric law describes the variation of Earth's pressure with respect to altitude:**

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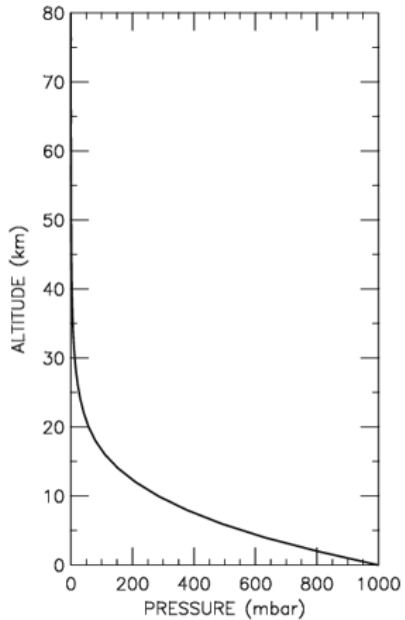
$$\text{Can show } H = R_{\text{EARTH}} T(z) / \text{grav} ,$$

$$\text{Since } R_{\text{EARTH}} = 2.88 \times 10^6 \text{ ergs / K gm}$$

$$\text{grav} = 981 \text{ cm sec}^{-2} \quad \text{and } T(\text{lower trop}) \approx 272 \text{ K}$$

$$\text{then } H(z=0) = 8.0 \times 10^5 \text{ cm} = 8 \text{ km}$$

# Pressure versus Altitude



In modern atmospheric sciences, the most handy version of the Ideal Gas Law is:

$$p = M k T$$

where  $p$  is pressure (force per unit area),  $M$  is number density (molecules/volume),  $k$  is Boltzmann's constant ( $1.38 \times 10^{-16}$  ergs/K), and  $T$  is temperature.

If  $p$  is given in units of mbar (or hPa),  $M$  is in units of  $\frac{\text{molecules}}{\text{cm}^3}$ , and  $T$  is in K,

then can show  $k$  must be  $1.38 \times 10^{-19} \frac{\text{mbar cm}^3}{\text{K molecules}}$

- **Barometric law describes the variation of Earth's pressure with respect to altitude:**

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where  $H$  is called the "scale height"

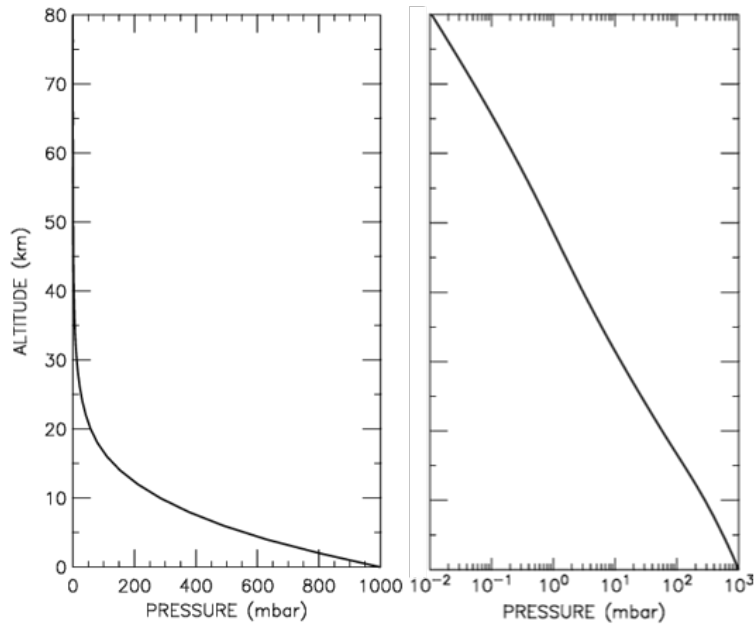
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# Pressure versus Altitude

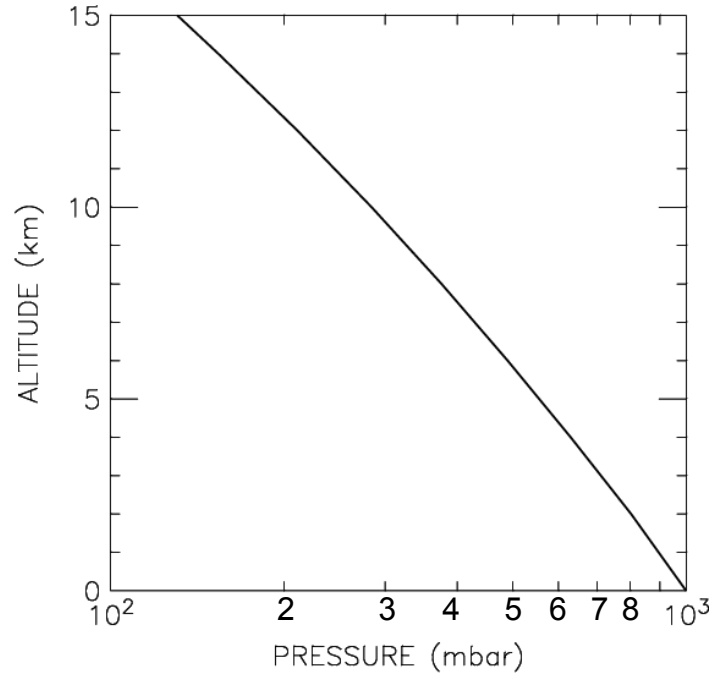


- **Barometric law describes the variation of Earth's pressure with respect to altitude:**

$$\text{Pressure (z)} = \text{Pressure (surface)} \times e^{-z/H}$$

**Two plots convey the same information !**

# Pressure versus Altitude



$$p(z=0) = 1013.25 \text{ mbar}$$
$$p(z=8 \text{ km}) =$$

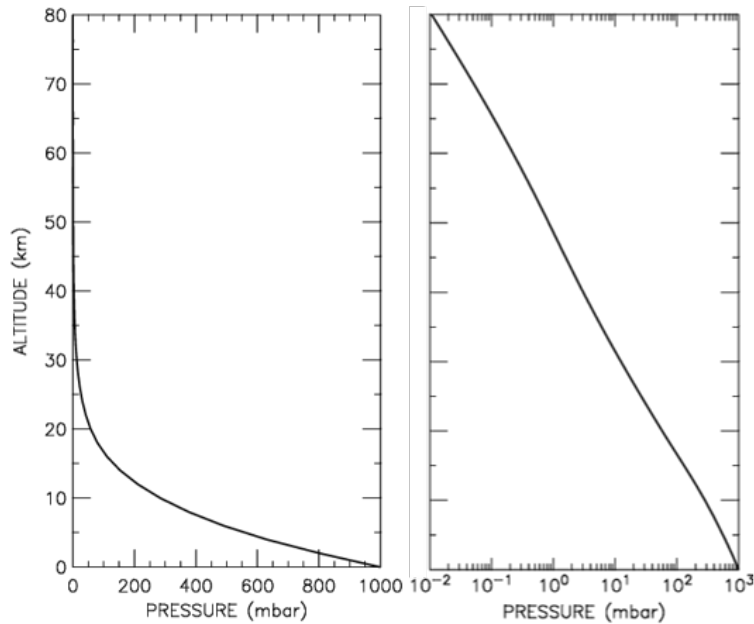
- **Barometric law describes the variation of Earth's pressure with respect to altitude:**

$$\text{Pressure}(z) = \text{Pressure}(\text{surface}) \times e^{-z/H}$$

**Let's take a closer look at log pressure versus altitude, in the troposphere**



# Pressure versus Altitude

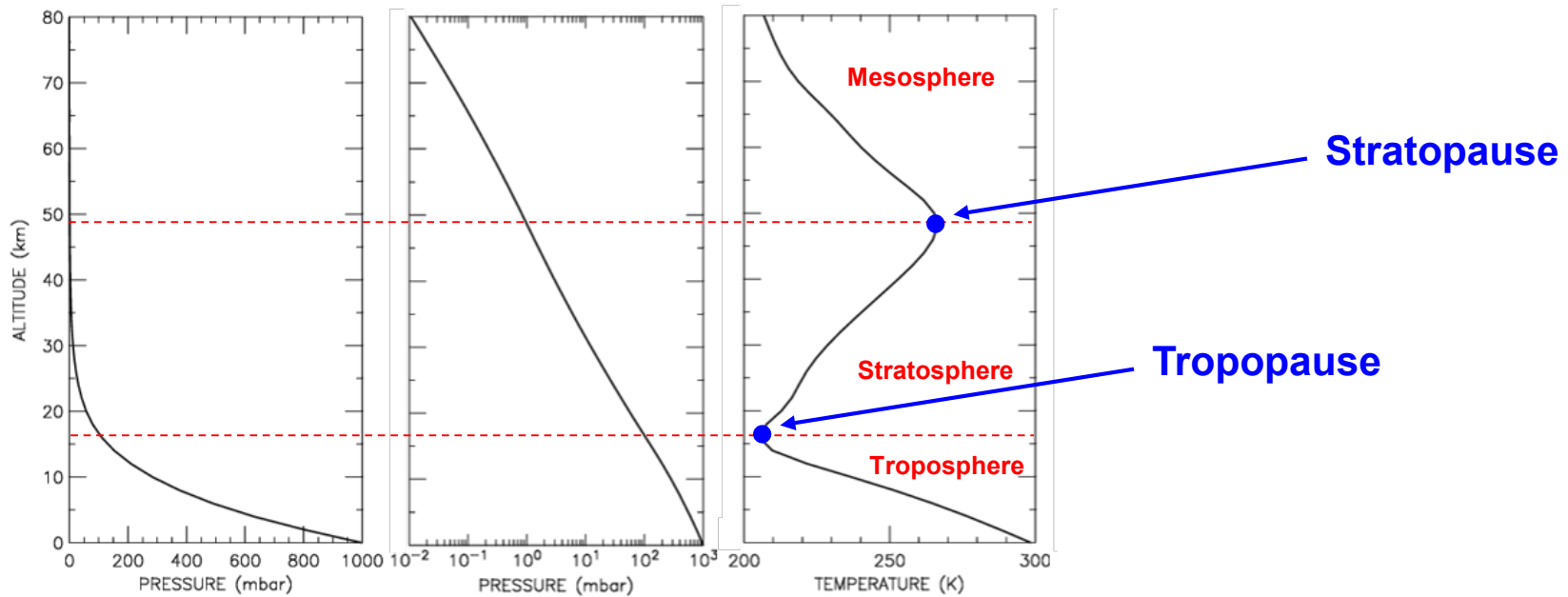


- **Barometric law** describes the variation of Earth's pressure with respect to altitude:

$$\text{Pressure (z)} = \text{Pressure (surface)} \times e^{-z/H}$$

**How does pressure vary as a function of depth, in the ocean ?**

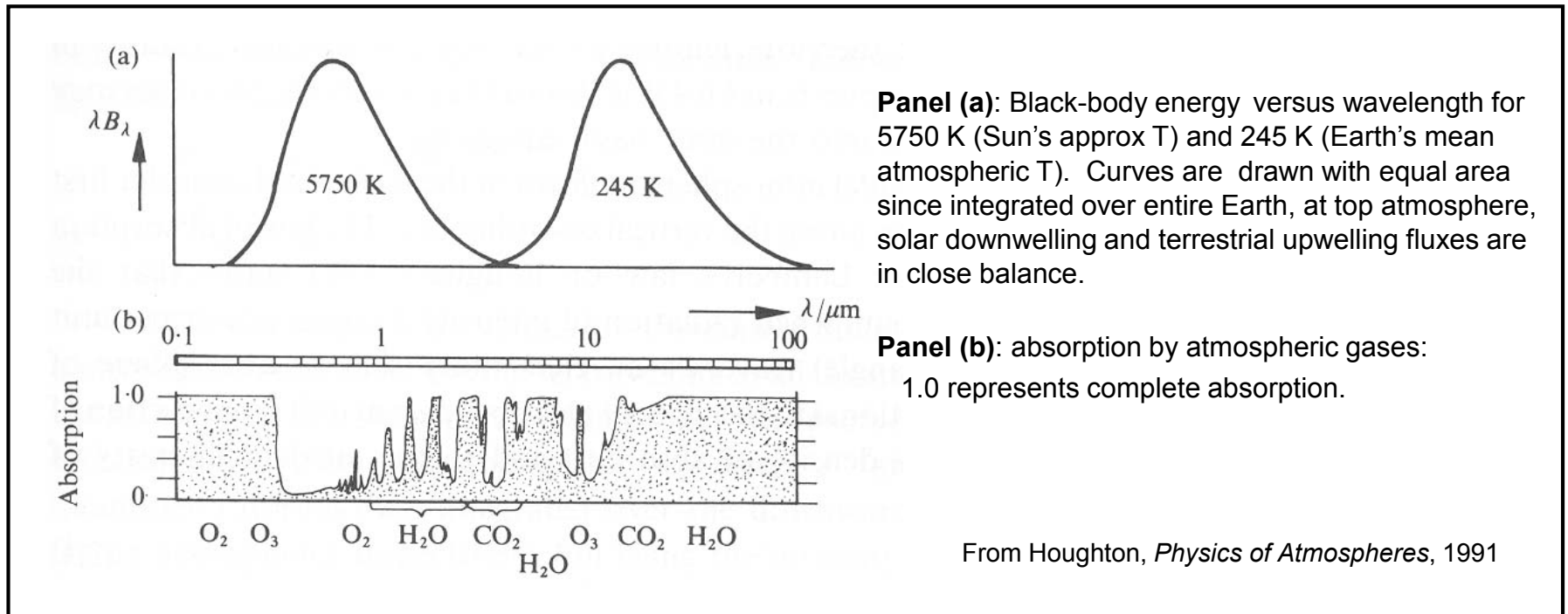
# Temperature versus Altitude



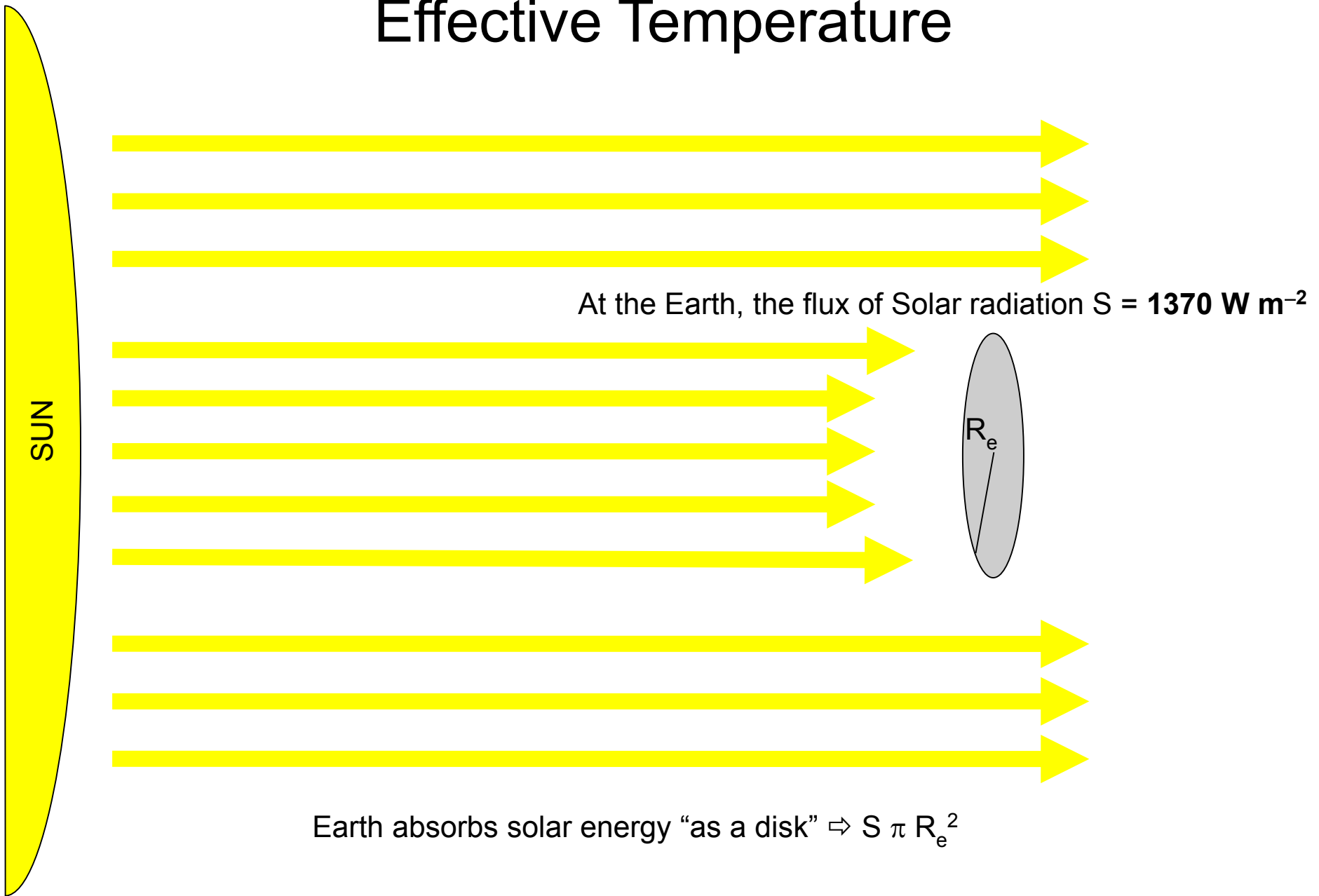
- **T falls wrt increasing altit until the tropopause, then rises wrt altit until the stratopause, then falls wrt to rising altitude**

# Atmospheric Radiation

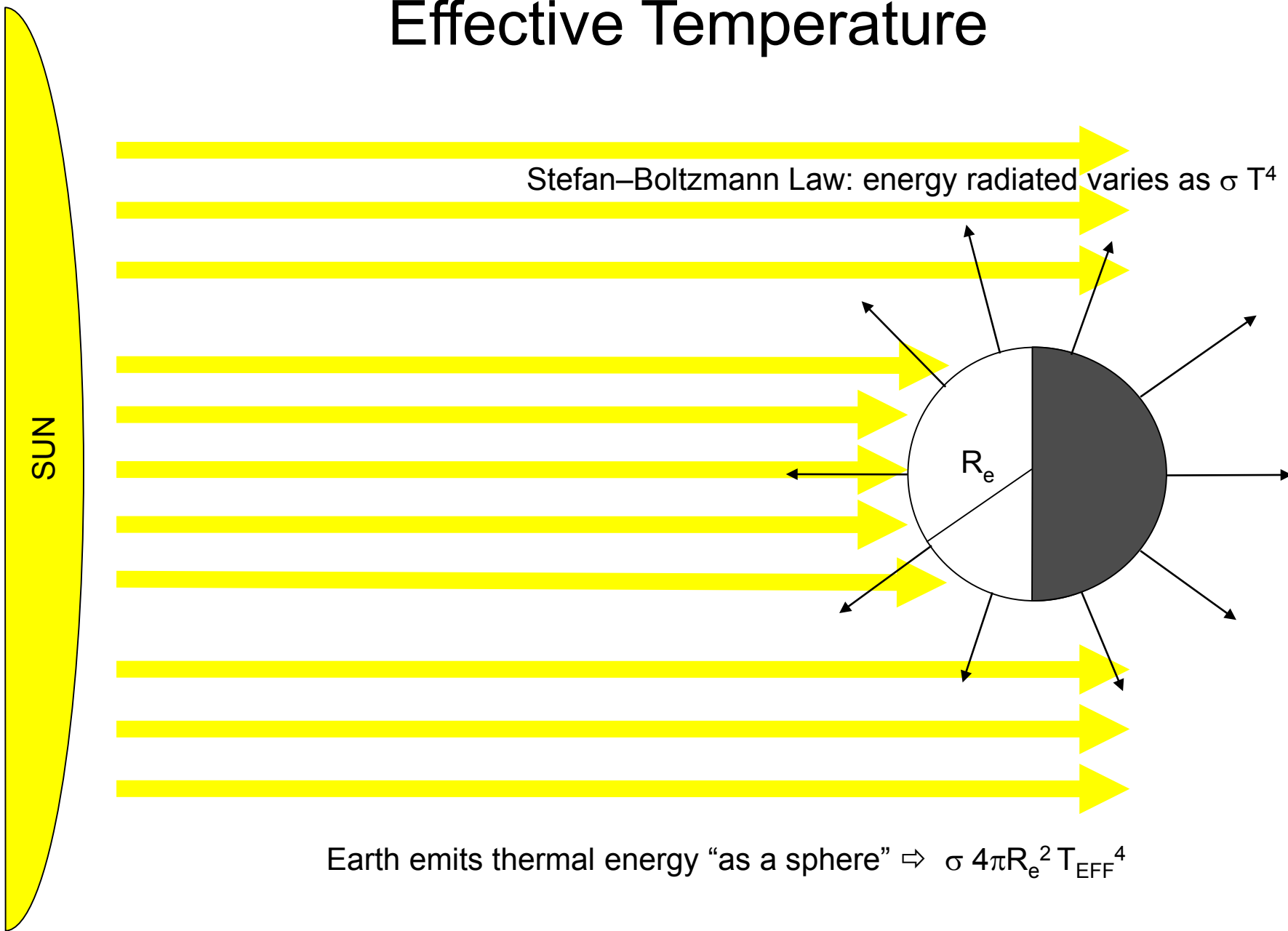
- Solar irradiance (downwelling) at top of atmosphere occurs at wavelengths between ~200 and 2000 nm (~5750 K “black body” temperature)
- Thermal irradiance (upwelling) at top of the atmosphere occurs at wavelengths between ~5 and 50  $\mu\text{m}$  (~245 K “black body” temperature)



# Effective Temperature



# Effective Temperature



# Effective Temperature

Earth absorbs solar energy “as a disk”  $\Rightarrow S \pi R_e^2$

Earth emits thermal energy “as a sphere”  $\Rightarrow \sigma 4\pi R_e^2 T_{\text{EFF}}^4$

$$S \pi R_e^2 = \sigma 4\pi R_e^2 T_{\text{EFF}}^4$$

or

$$S = 4 \sigma T_{\text{EFF}}^4$$

Group Quiz #1:

Above formula lacks the term that represents Earth’s albedo  
Find  $T_{\text{EFF}}$  for Earth, using Earth’s albedo, for:

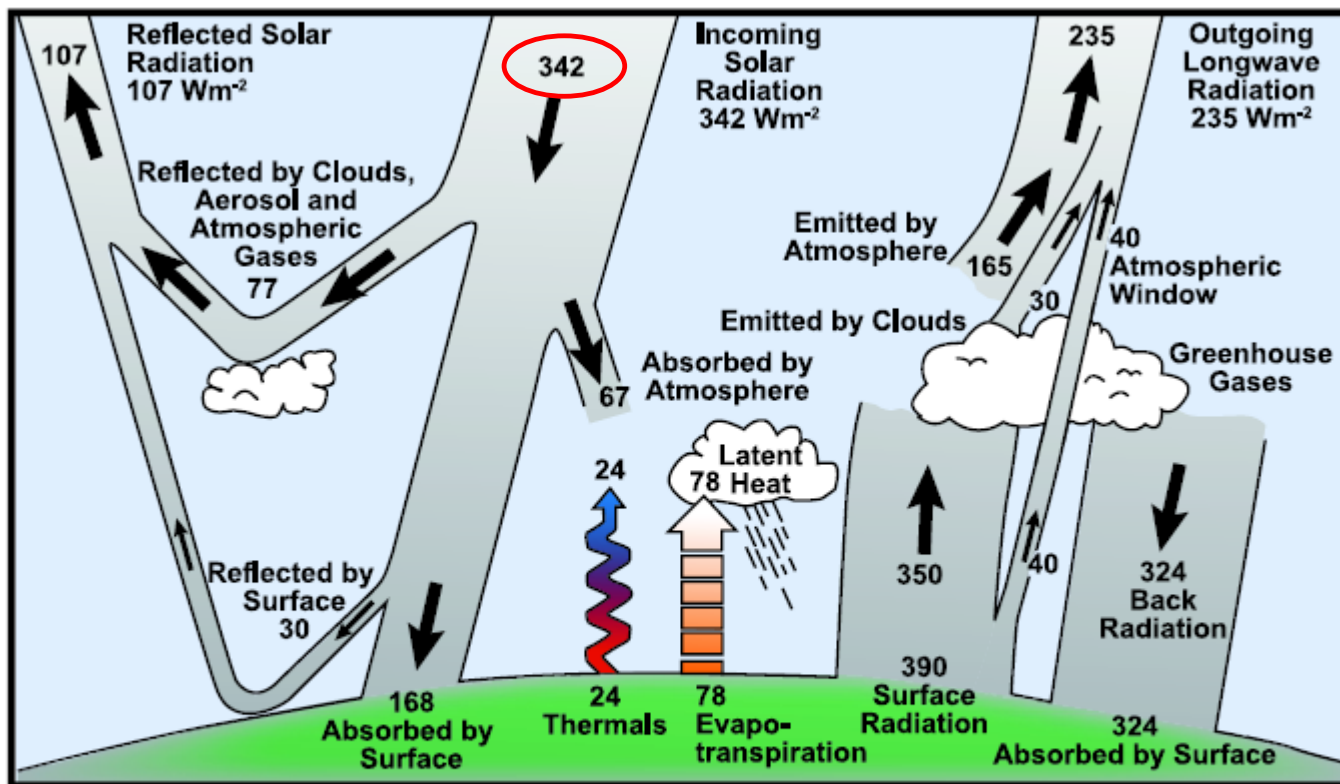
$$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

$$S = 1370 \text{ W m}^{-2}$$

$$\text{Albedo} = 0.3$$

# Effective Temperature

Let's take a closer look at  $S = 1370 \text{ W m}^{-2}$



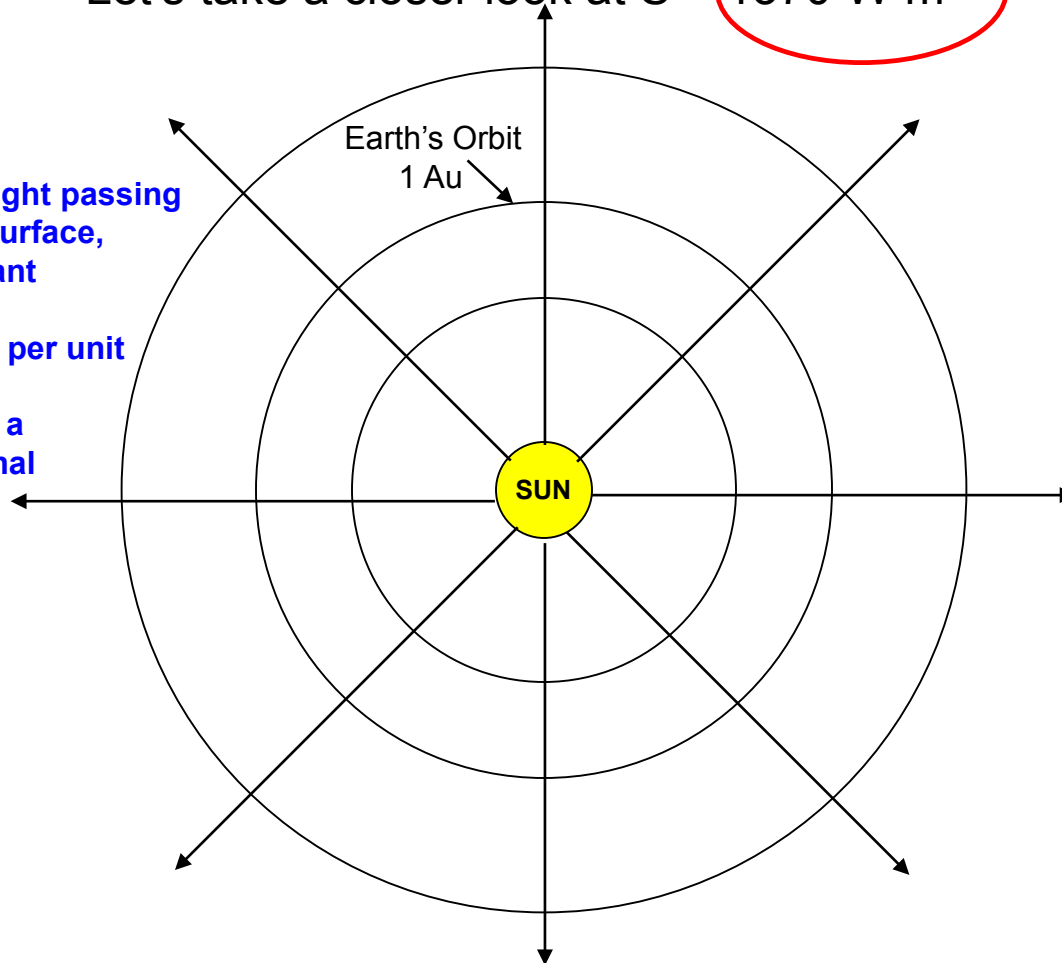
FAQ 1.1, Figure 1. Estimate of the Earth's annual and global mean energy balance. Over the long term, the amount of incoming solar radiation absorbed by the Earth and atmosphere is balanced by the Earth and atmosphere releasing the same amount of outgoing longwave radiation. About half of the incoming solar radiation is absorbed by the Earth's surface. This energy is transferred to the atmosphere by warming the air in contact with the surface (thermals), by evapotranspiration and by longwave radiation that is absorbed by clouds and greenhouse gases. The atmosphere in turn radiates longwave energy back to Earth as well as out to space. Source: Kiehl and Trenberth (1997).

# Effective Temperature

Let's take a closer look at  $S = 1370 \text{ W m}^{-2}$

The total amount of sunlight passing through each spherical surface, of various radii, is constant

Therefore the energy (W) per unit area ( $\text{m}^{-2}$ ) decreases wrt Distance from the Sun in a manner that is proportional to: \_\_\_\_\_

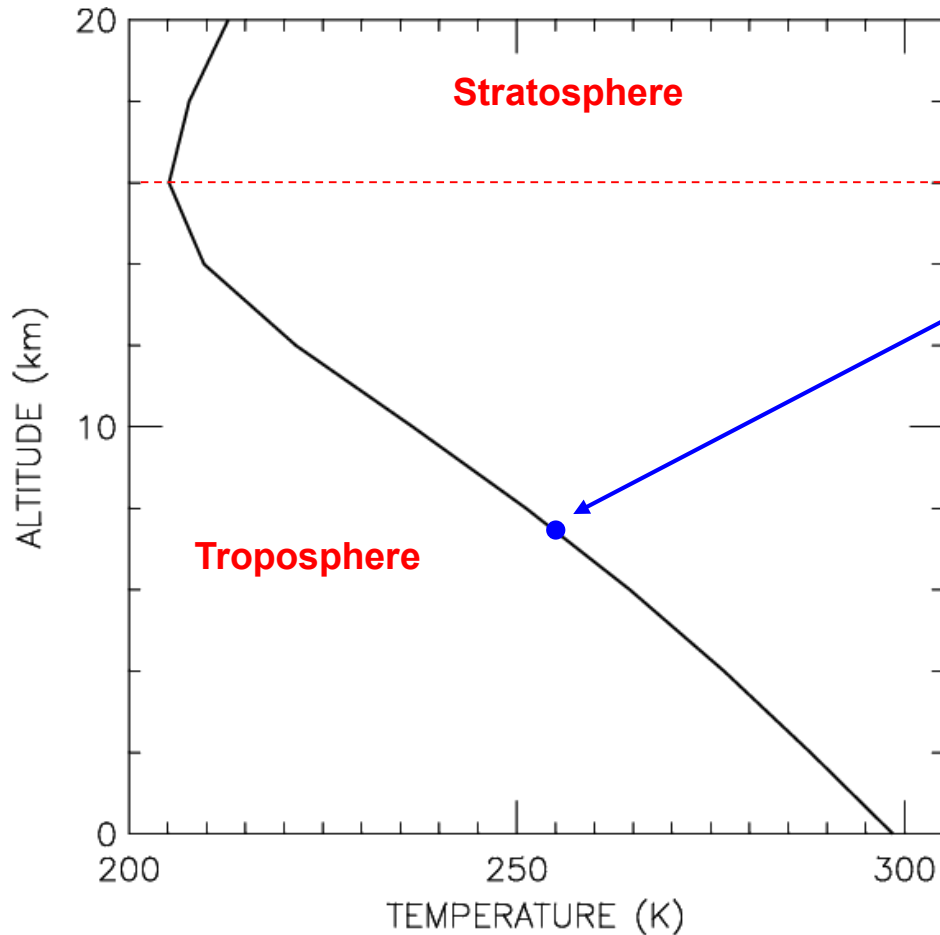


- Notes: 1) Au, or Astronomical Unit, is a measure of the distance of a planet from the Sun, normalized by the mean distance of Earth from the Sun. So by definition, Earth's orbit is 1 Au from the Sun
- 2) The diagram above represents orbits as perfect spheres, which is suitable for our study of effective temperatures. In reality, of course, planets orbit the Sun in an elliptical manner.



# Temperature versus Altitude

Let's take a closer look at  $T_{\text{EFF}} =$

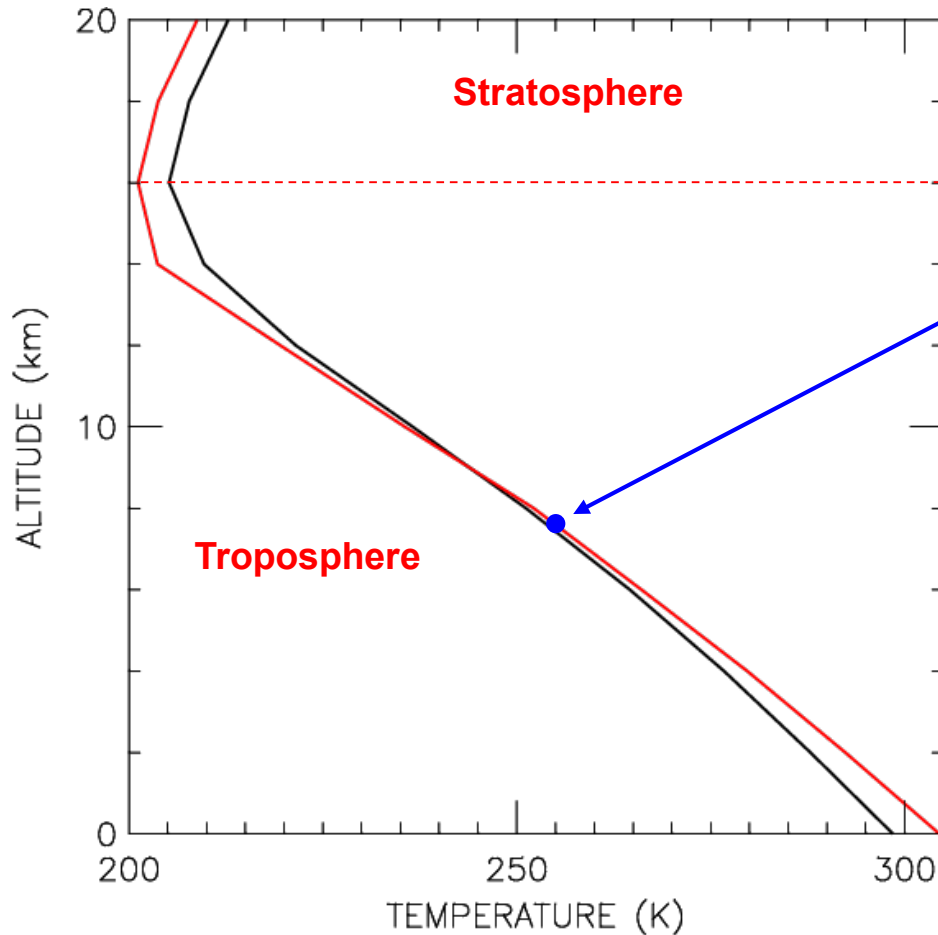


Altitude in troposphere where  
 $T =$

Termed Earth's mean radiating  
altitude to space

# Temperature versus Altitude

Let's take a closer look at  $T_{\text{EFF}} =$



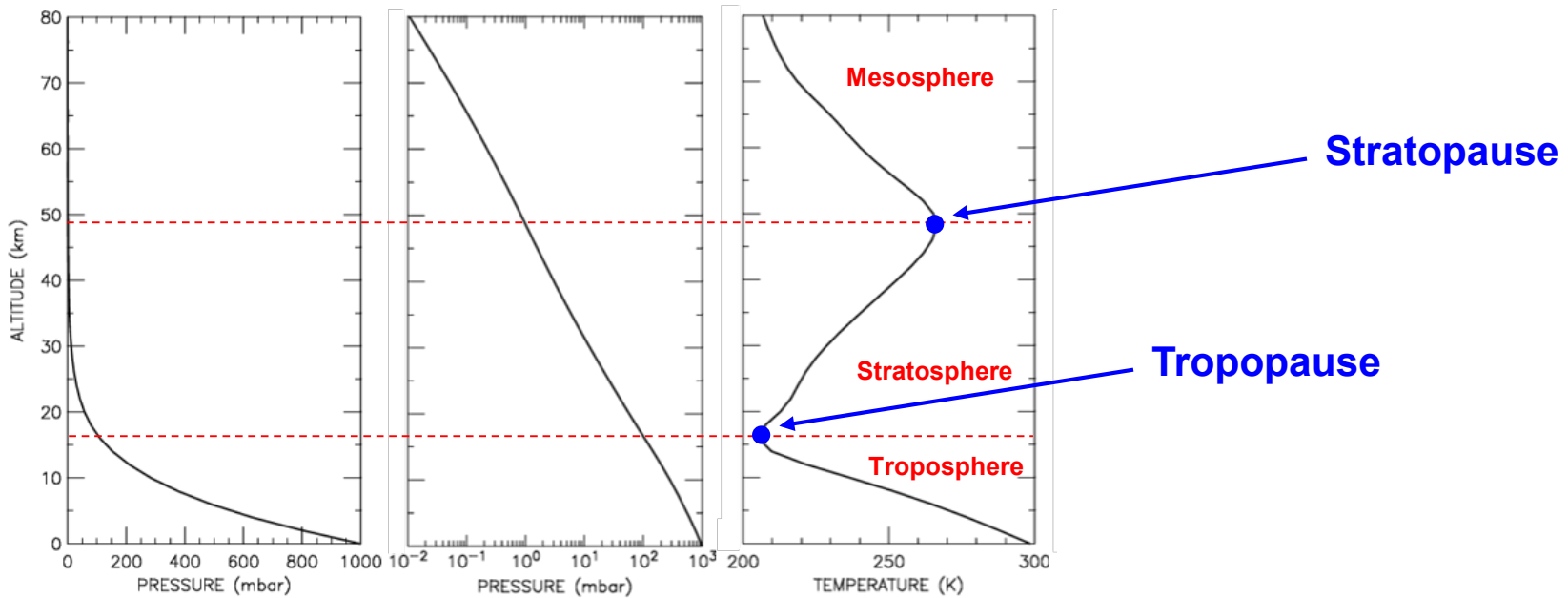
Altitude in troposphere where  $T =$

Termed Earth's mean radiating altitude to space

As Earth warms in response to rising GHGs, the lower trop will warm, the strat will cool, and the mean radiating altitude will likely rise slightly higher

Regardless, the  $T$  of the mean radiating altitude will not change

# Temperature versus Altitude



- **T falls wrt increasing altit until the tropopause, then rises wrt altit until the stratopause, then falls wrt to rising altitude**

If the troposphere is dry,  $dT/dz = - grav / c_p$

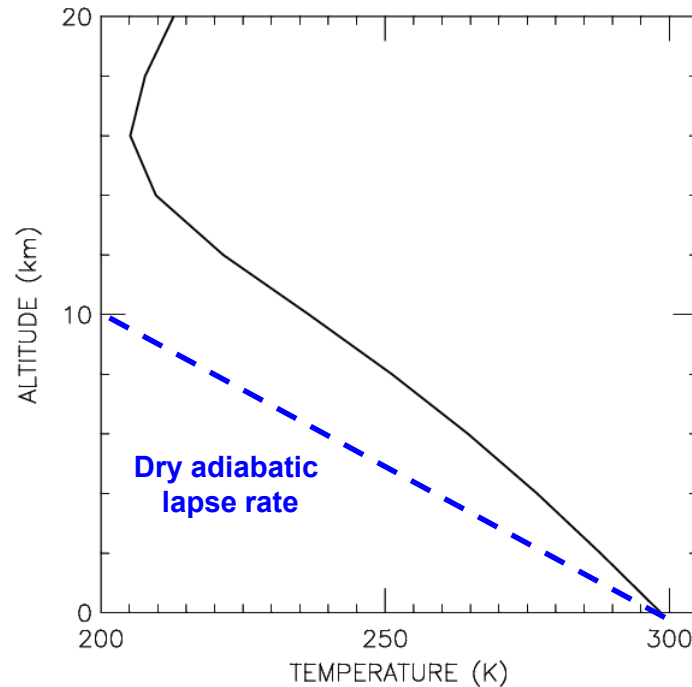
where  $c_p$  is specific heat of air at constant pressure =  $1 \times 10^7 \text{ erg gm}^{-1} \text{ K}^{-1}$

Note:  $1 \text{ erg} = 1 \text{ dyne cm} = \text{gm cm}^2 \text{ sec}^{-2}$

$$\Rightarrow dT/dz^{\text{DRY}} = - 981 \text{ cm sec}^{-2} / (10^7 \text{ cm}^2 \text{ sec}^{-2} \text{ K}^{-1}) \times 10^5 \text{ cm/km} = 9.8 \text{ K / km}$$

**Dry adiabatic lapse rate**

# Temperature versus Altitude



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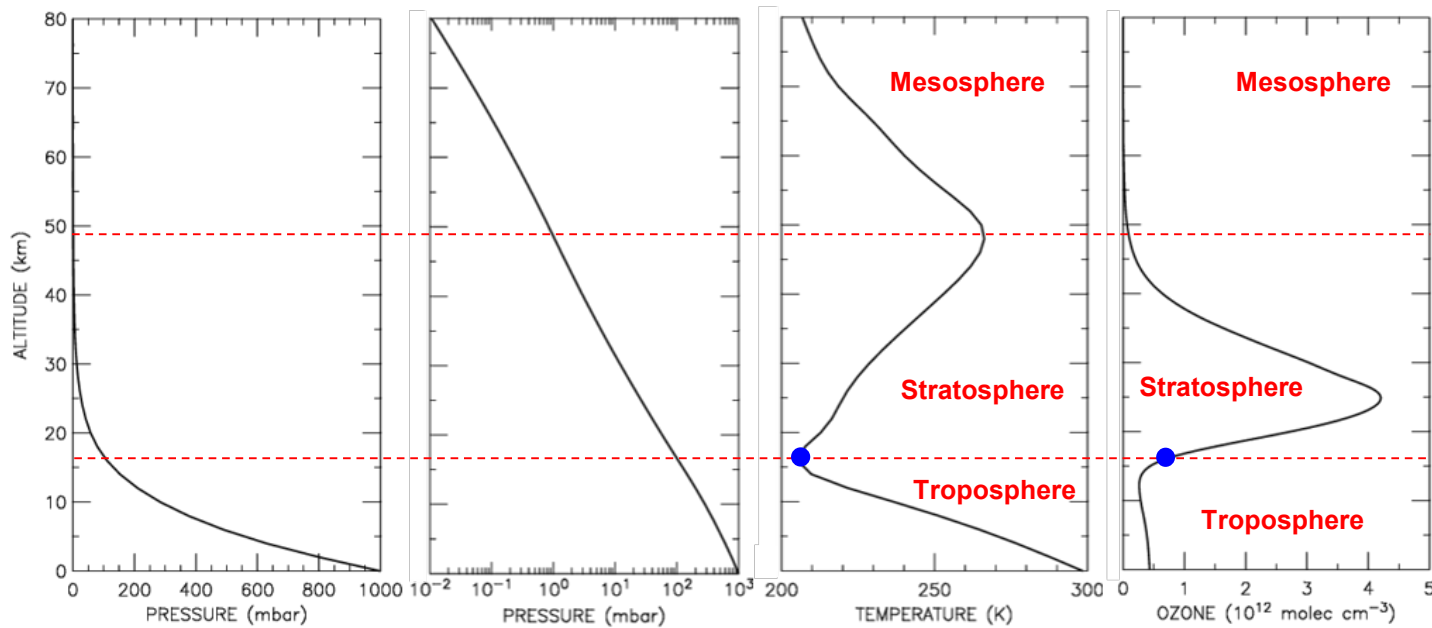
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**Dry adiabatic lapse rate**

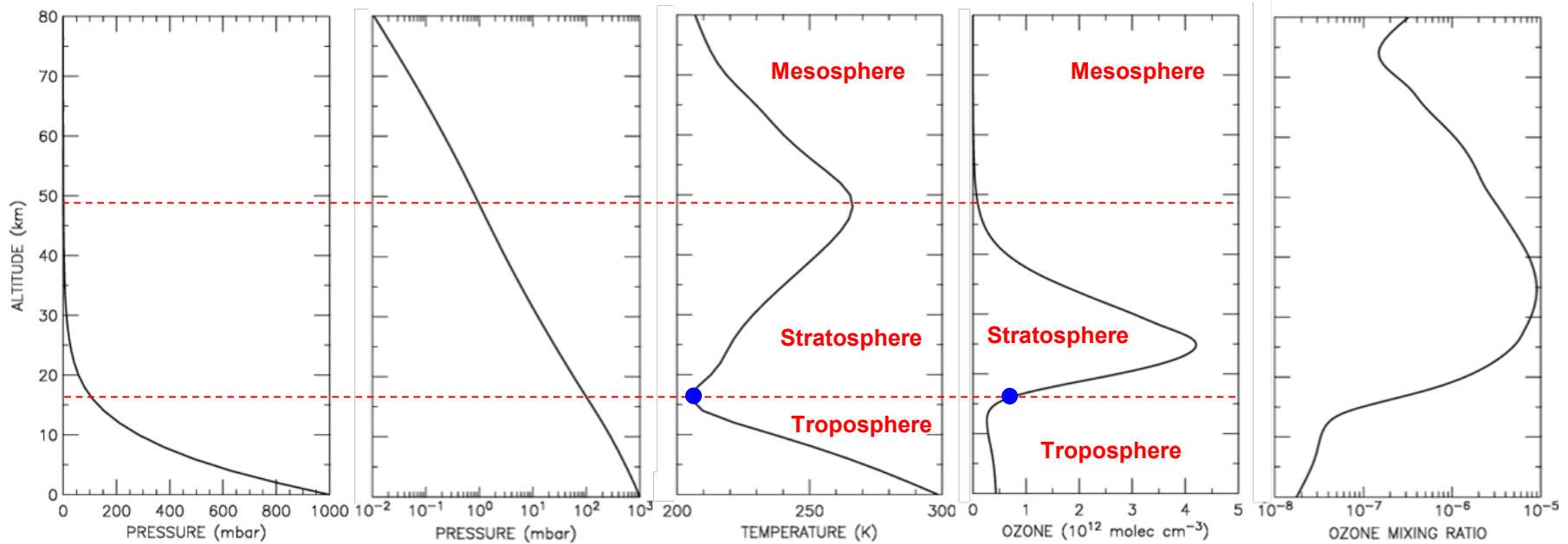
# Temperature versus Altitude



- **T falls wrt increasing altit until the tropopause, then rises wrt altit until the stratopause, then falls wrt to rising altitude**

**Fourth chart expresses abundance of ozone concentration, or ozone density, or  $[\text{O}_3]$ , in units of molecules /  $\text{cm}^3$**

# Temperature versus Altitude



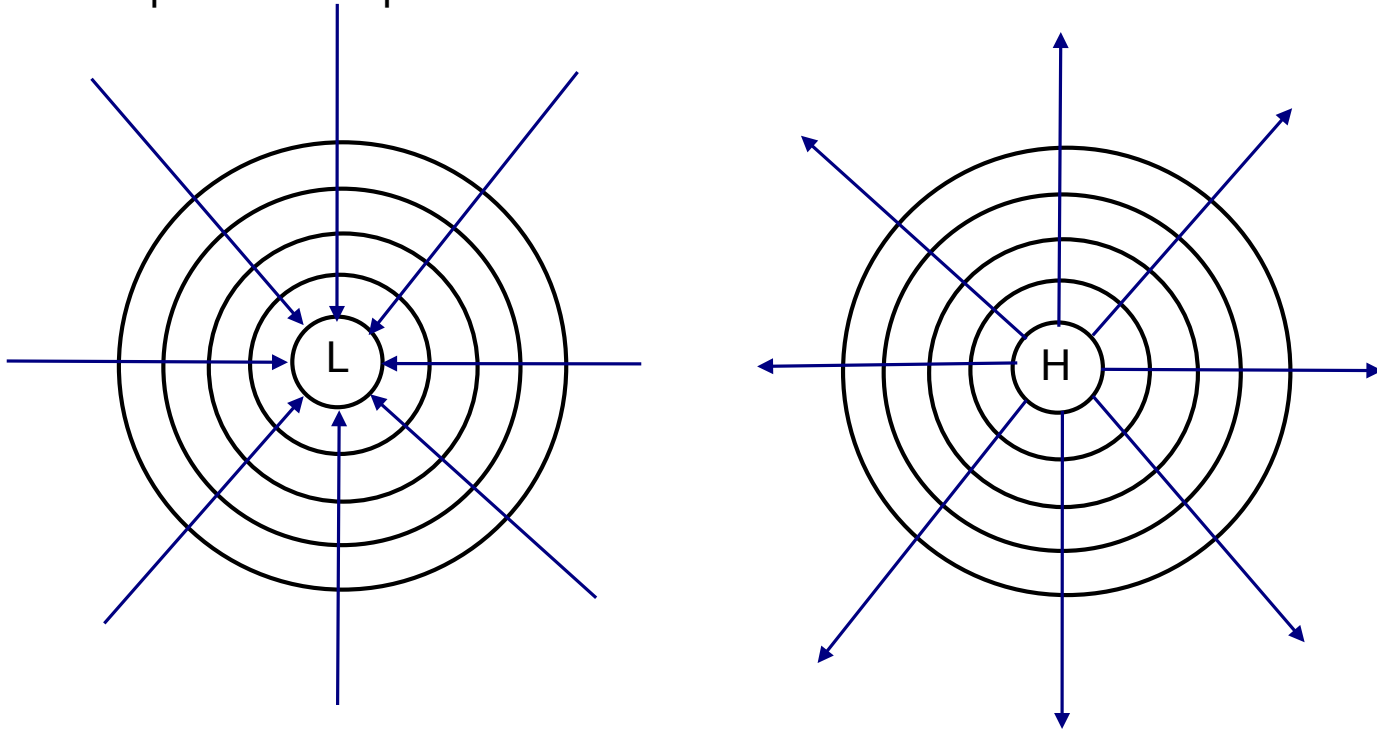
- **T falls wrt increasing altit until the tropopause, then rises wrt altit until the stratopause, then falls wrt to rising altitude**

**Chart on far right expresses ozone mixing ratio,  $O_3$  mr in dimensionless units, where  $O_3$  mr =  $[O_3] / [M]$ , where  $[M]$  is the concentration (or density) of air**

# Coriolis Force

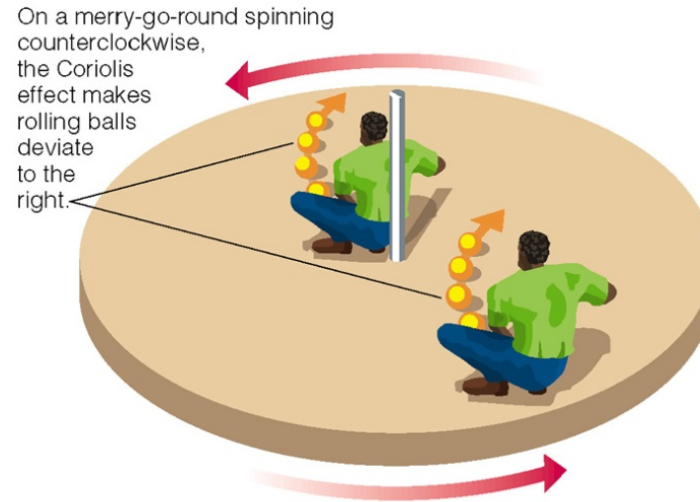
So far, we've reviewed temperature, pressure, and the balance between solar energy input to the atmosphere and terrestrial radiation leaving the atmosphere.

There's one more piece of the puzzle that we need to be familiar with.



In general, air moves from areas of high pressure to areas of low pressure. In the absence of external forces, air will move in a straight line, following pressure gradients

# Coriolis Force



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<http://lasp.colorado.edu/~bagenal/3720/CLASS15/15EVM-Dyn1.html>

Earth's rotation provides an apparent force that deflects air  
to the right in the Northern Hemisphere,  
to the left in the Southern Hemisphere.

Force is proportional to  $\sin(\text{latitude})$ , so vanishes at the equator



# Geostrophy

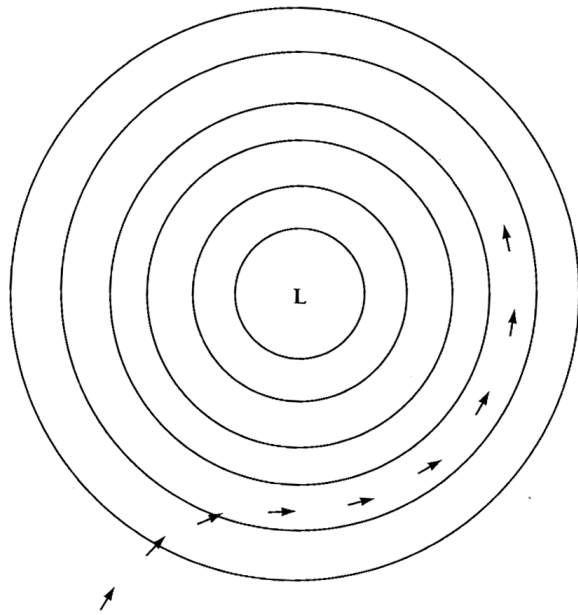


Figure 8.16 Track of an air parcel in the vicinity of a low pressure region in the Northern Hemisphere. The parcel is initially at rest but then adjusts to the pressure gradient force and the Coriolis force to achieve geostrophic balance.

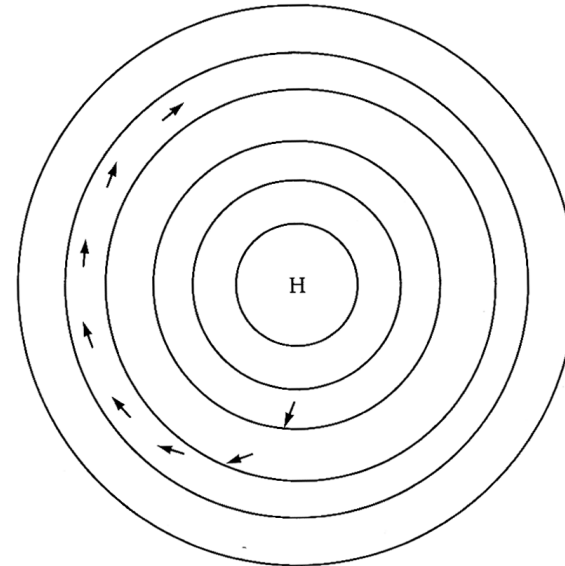


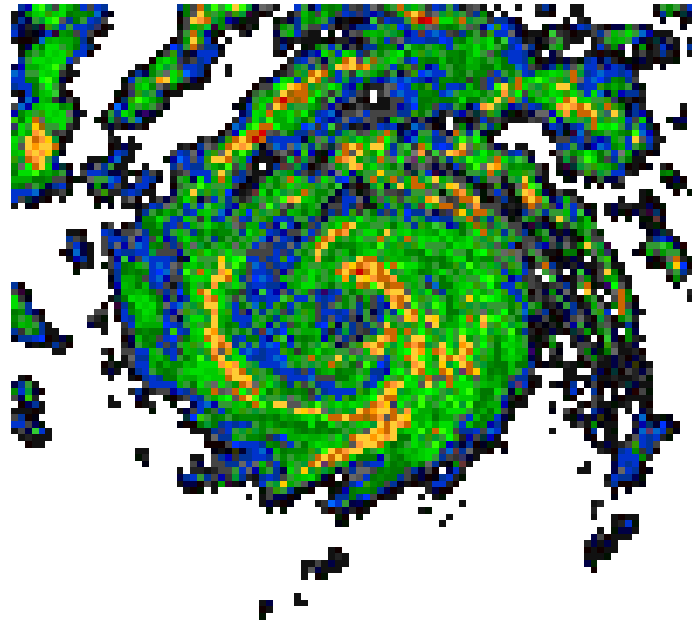
Figure 8.17 Same situation as in 8.16, except that the parcel is in the vicinity of a high pressure region in the Northern Hemisphere.

From "The Atmospheric Environment", M. B. McElroy

**Geostrophic balance: balance between Coriolis Force and pressure gradient**

# Geostrophy

## NH Weather System:

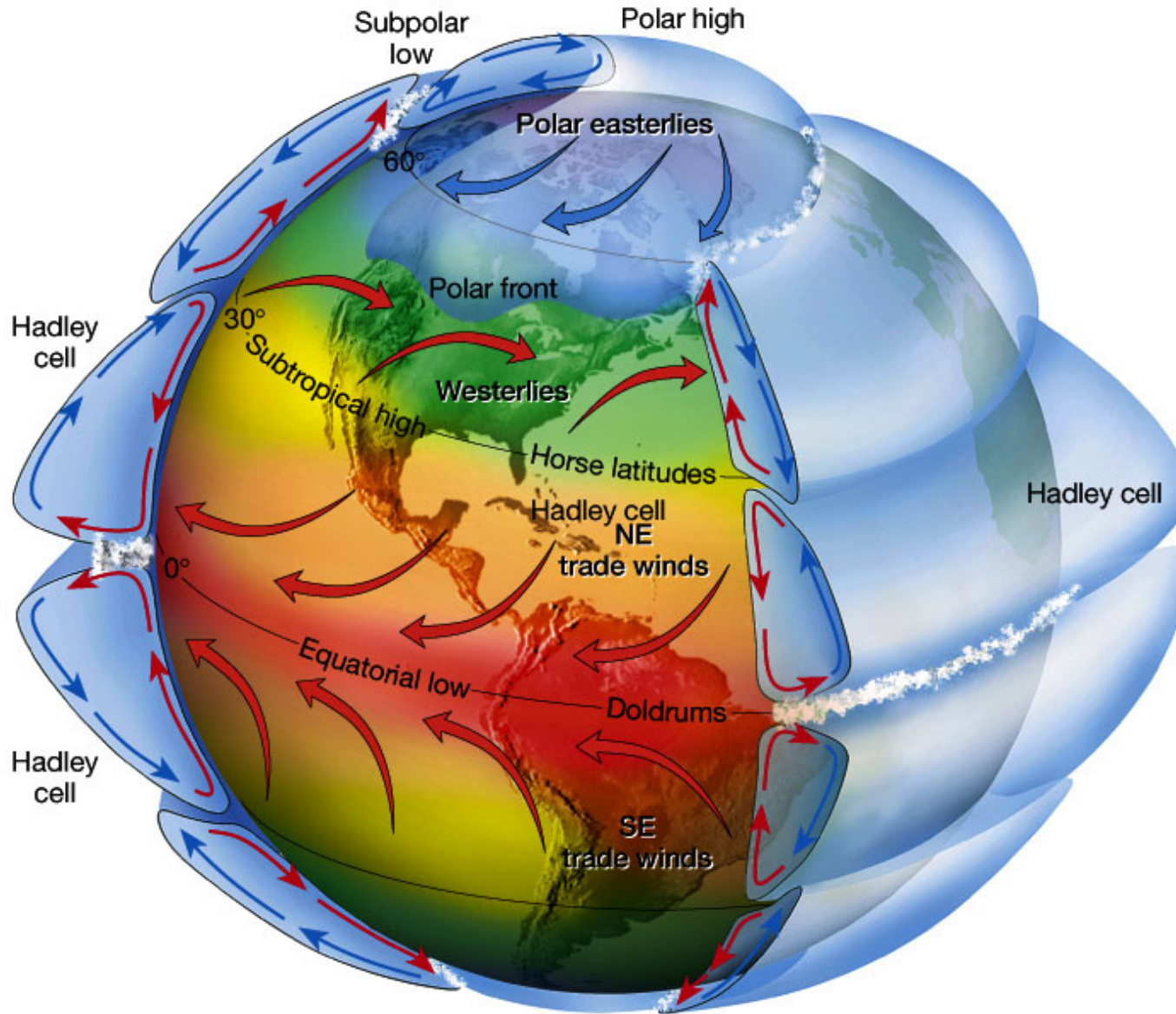


***Cyclonic Flow:*** when the wind swirls  
**counter-clockwise in the NH**

**Hurricane:** Cyclonic flow occurring in the N Atlantic or NE Pacific Ocean east of the dateline.

**Typhoon:** Cyclonic flow occurring over the NW Pacific Ocean, west of the dateline.

# Ferrel Circulation (Modern View)



<http://www.ux1.eiu.edu/~cfjps/1400/circulation.html>

# The Barometric Law

Assume a sample volume is at rest with respect to vertical motion :

$$p(z) - p(z + \Delta z) = \rho \text{ grav } \Delta z$$

in other words, the pressure difference between  $z$  and  $z + \Delta z$  is equal to the weight of air contained in a volume of unit horiz area.

Using calculus:

$$\frac{dp}{dz} = -\rho(z) \text{ grav}$$

Writing the gas law as  $p = R_{\text{EARTH}} \rho T$

$$\text{where } R_{\text{EARTH}} = 8.3143 \times 10^7 \frac{\text{ergs}}{\text{K mole}} \times \frac{\text{mole}}{28.8 \text{ gm}} = 2.87 \times 10^6 \text{ ergs/ K gm}$$

and substiting gives:

$$\frac{dp}{dz} = - \frac{p \text{ grav}}{R_{\text{EARTH}} T}$$

Or

$$\frac{dp}{p} = - \frac{dz}{H} \quad \text{where } H = \frac{R_{\text{EARTH}} T}{\text{grav}}$$

The solution of this ODE is:

$$p(z) = p(z = 0)e^{-z/H}$$