

Analysis Methods in Atmospheric and Oceanic Science

AOSC 652

Numerical Integration

Week 7, Day 3

- **Review HW #6**
- **General help with HW #7**

14 Oct 2016

AOSC 652: Analysis Methods in AOSC

Between now and Monday:

IDL Tract:

**Please read Chapters 1, 2, and 3 of Bowman
*An Introduction to Programming with IDL***

Python Tract:

**Please read Chapters 2, 3, 4, and 5 of DeCaria,
*Python Programming and Visualization for Scientists***

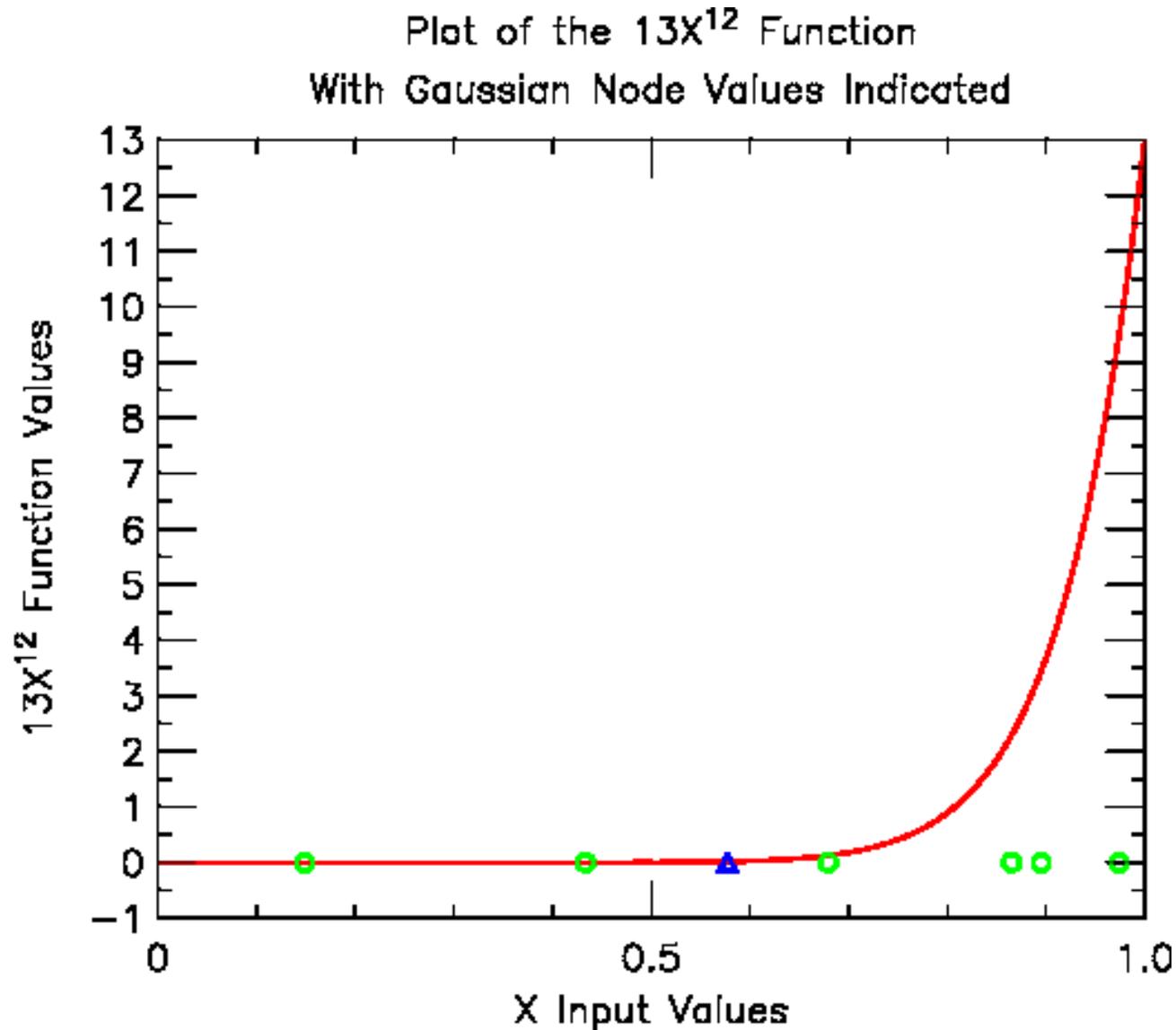
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Gaussian Integration

Function	2 Nodes	10 Nodes	Error, 10 Nodes
$3x^2$	1.000	1.000	-7.42×10^{-13}
$5x^4$	0.972	1.000	-8.18×10^{-13}
$9x^8$	0.674	1.000	-8.92×10^{-13}
$13x^{12}$	0.376	1.000	-8.29×10^{-13}

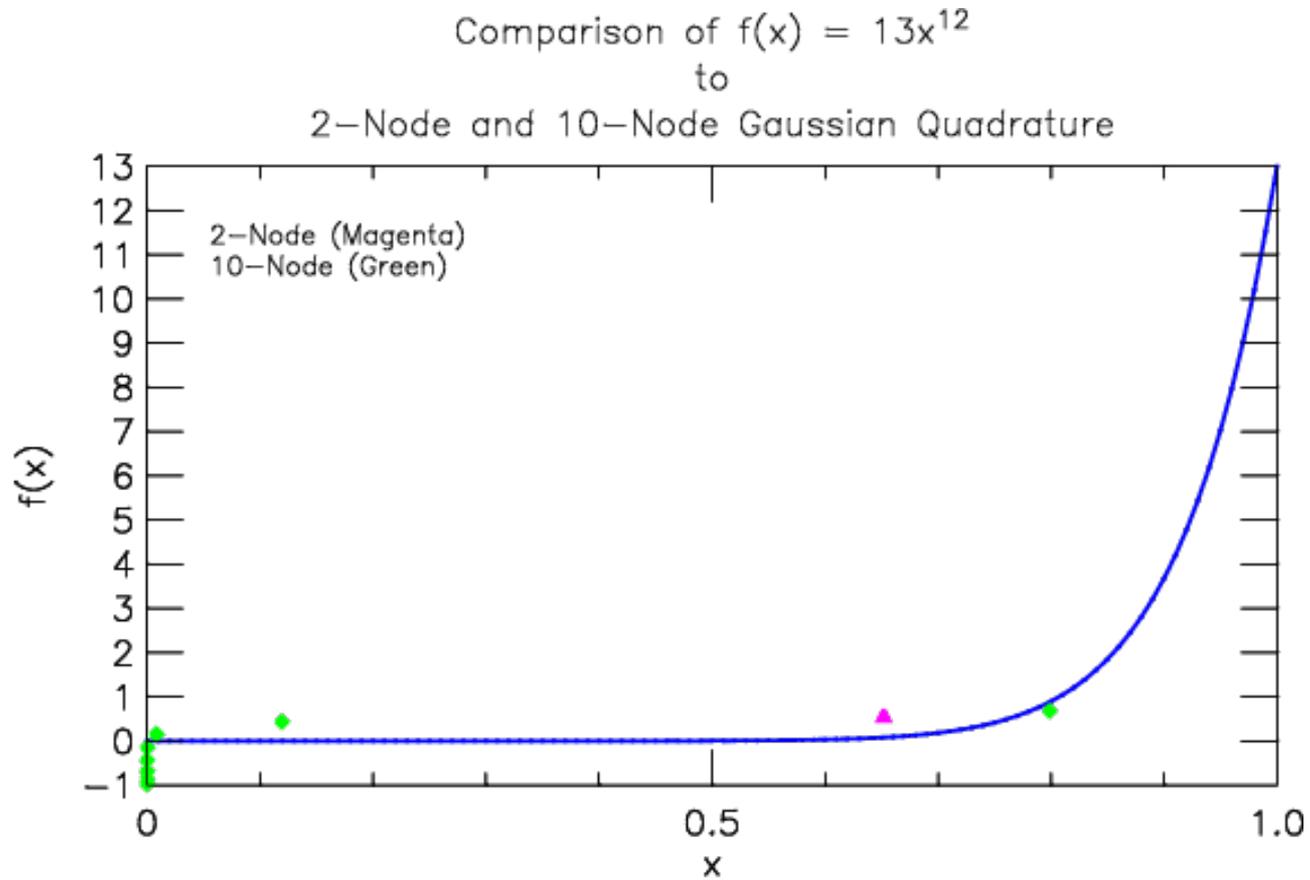
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Gaussian Integration



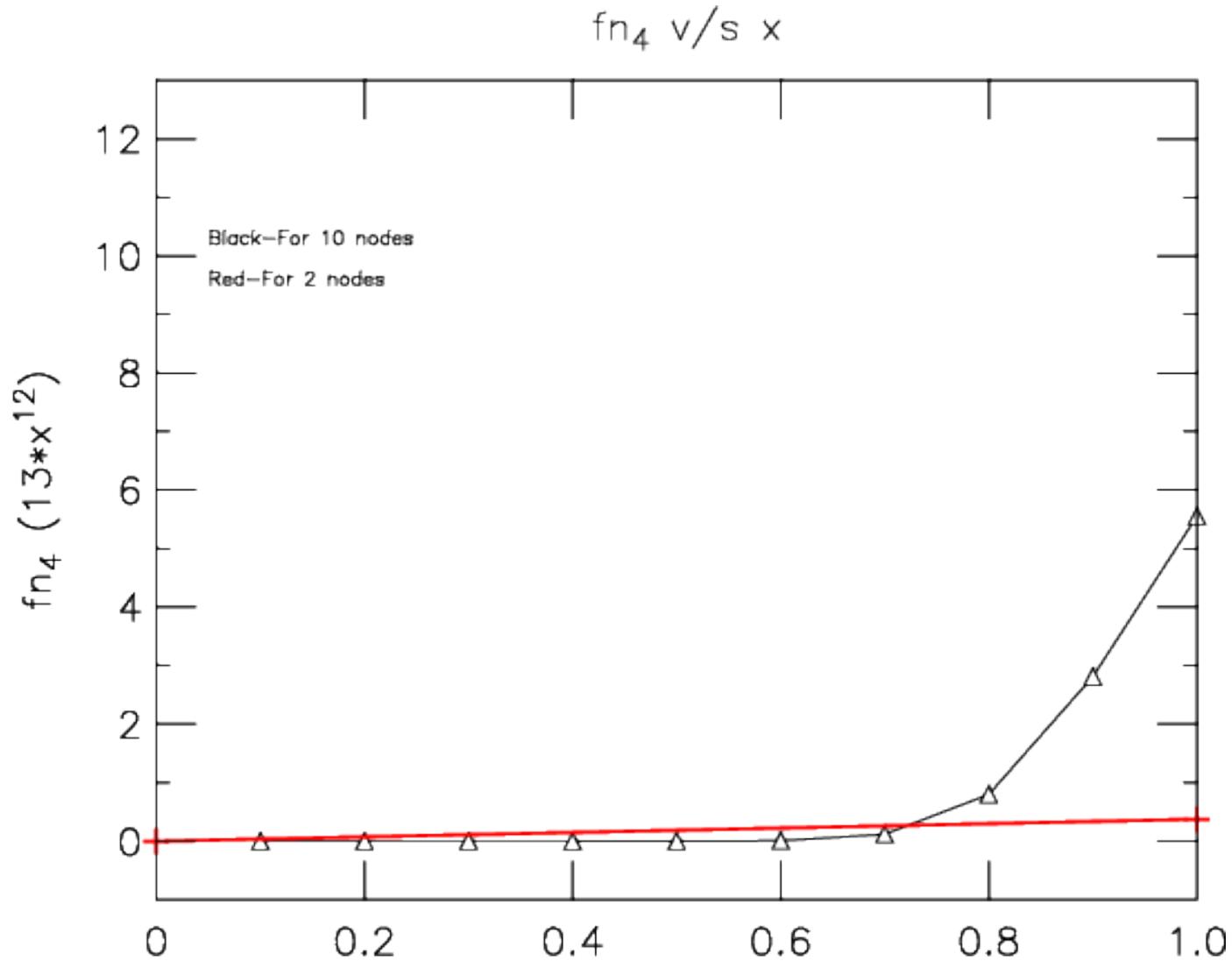
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Gaussian Integration



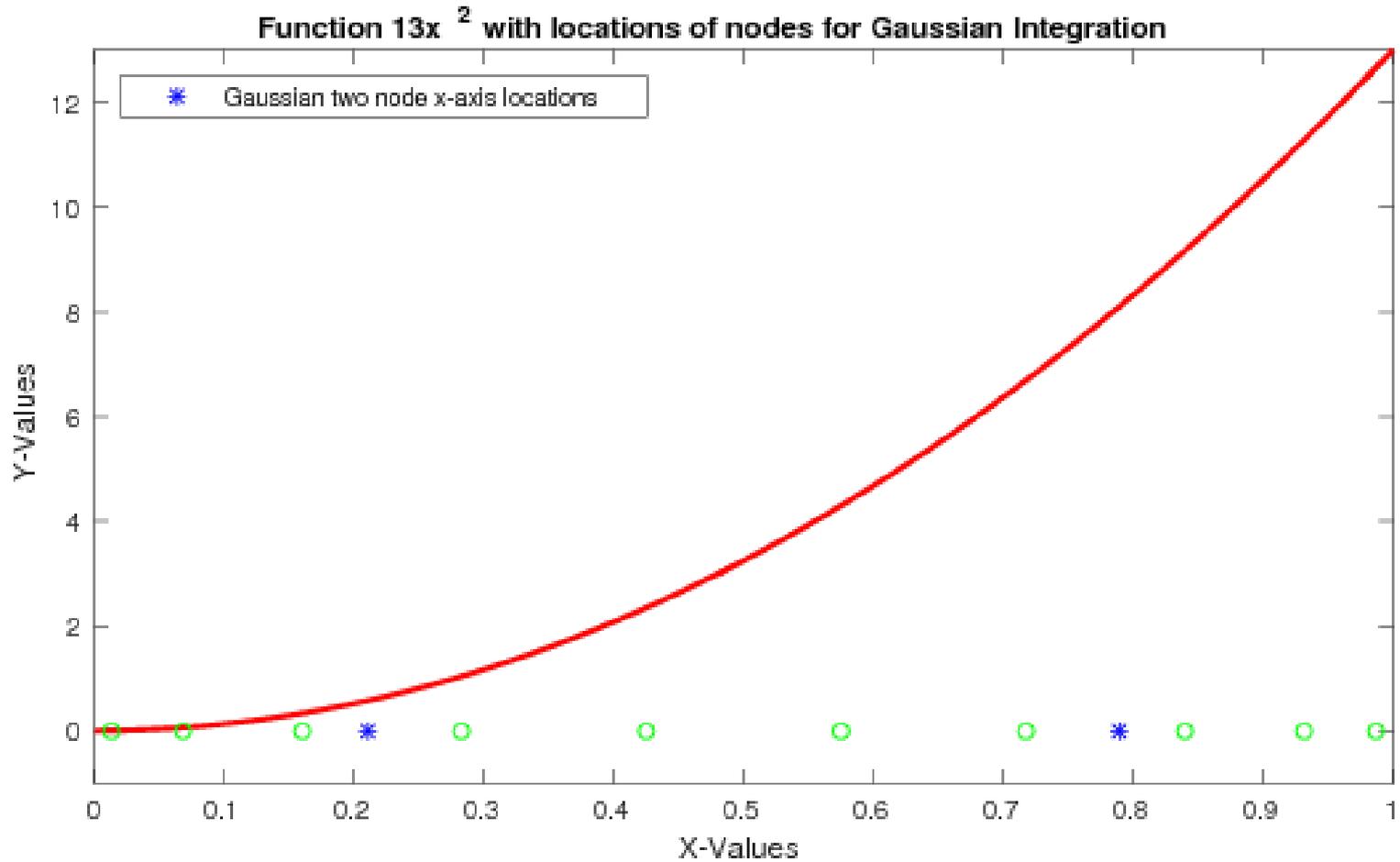
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Gaussian Integration



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Gaussian Integration



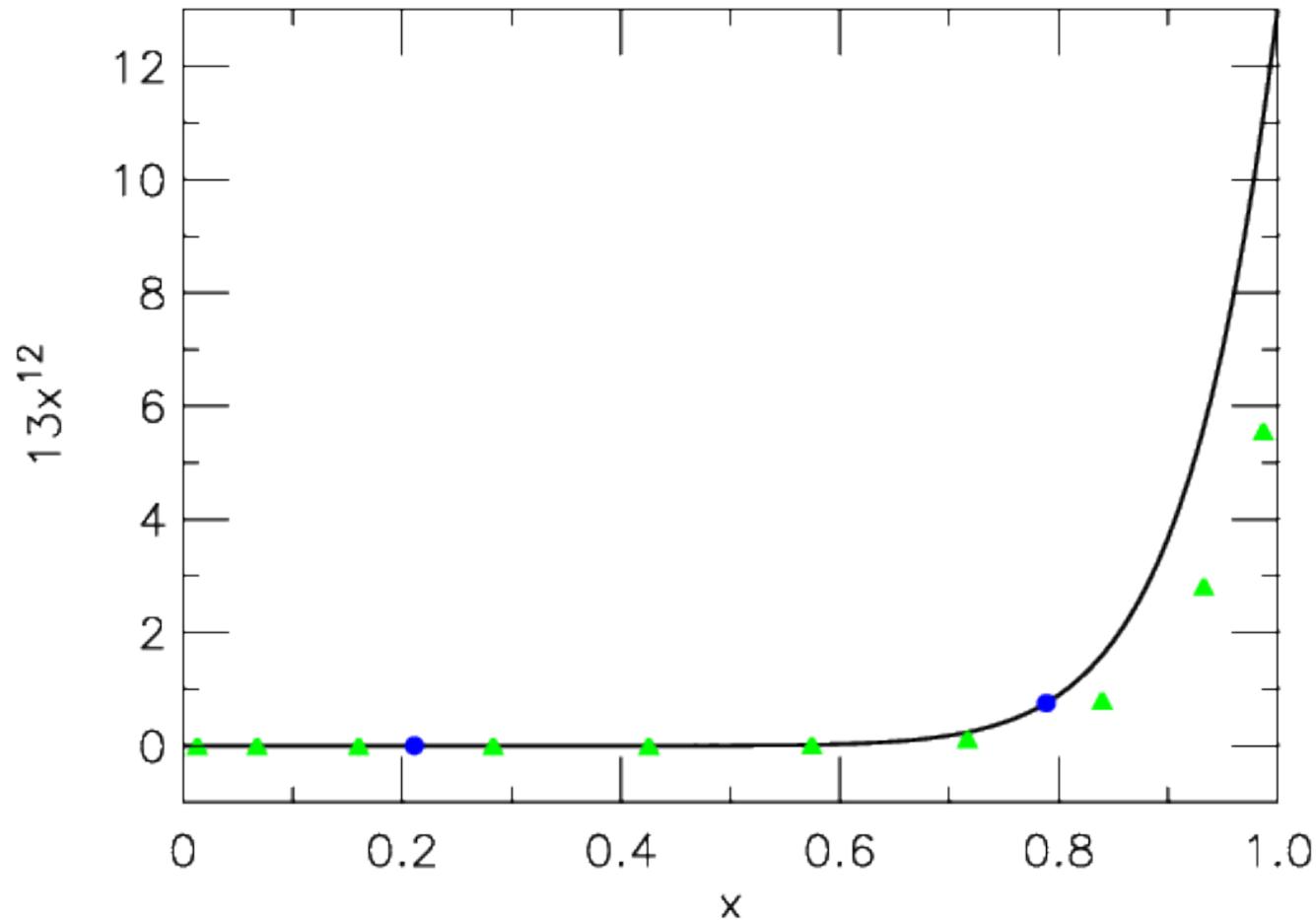
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Gaussian Integration

Black: $13x^{12}$, intervals = 100

Green: Evaluation of 10 node Gaussian integration

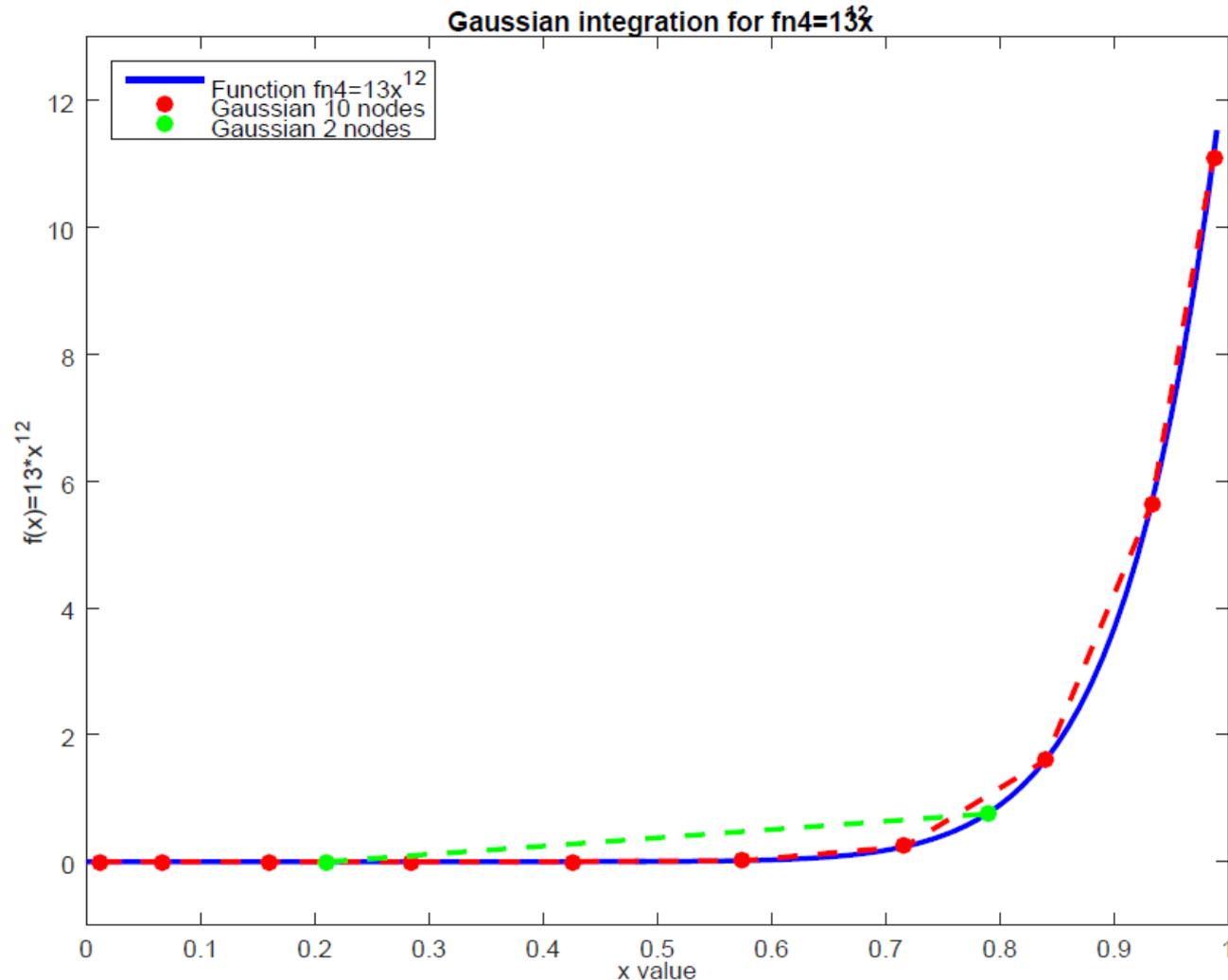
Blue: Evaluation of 2 node Gaussian integration



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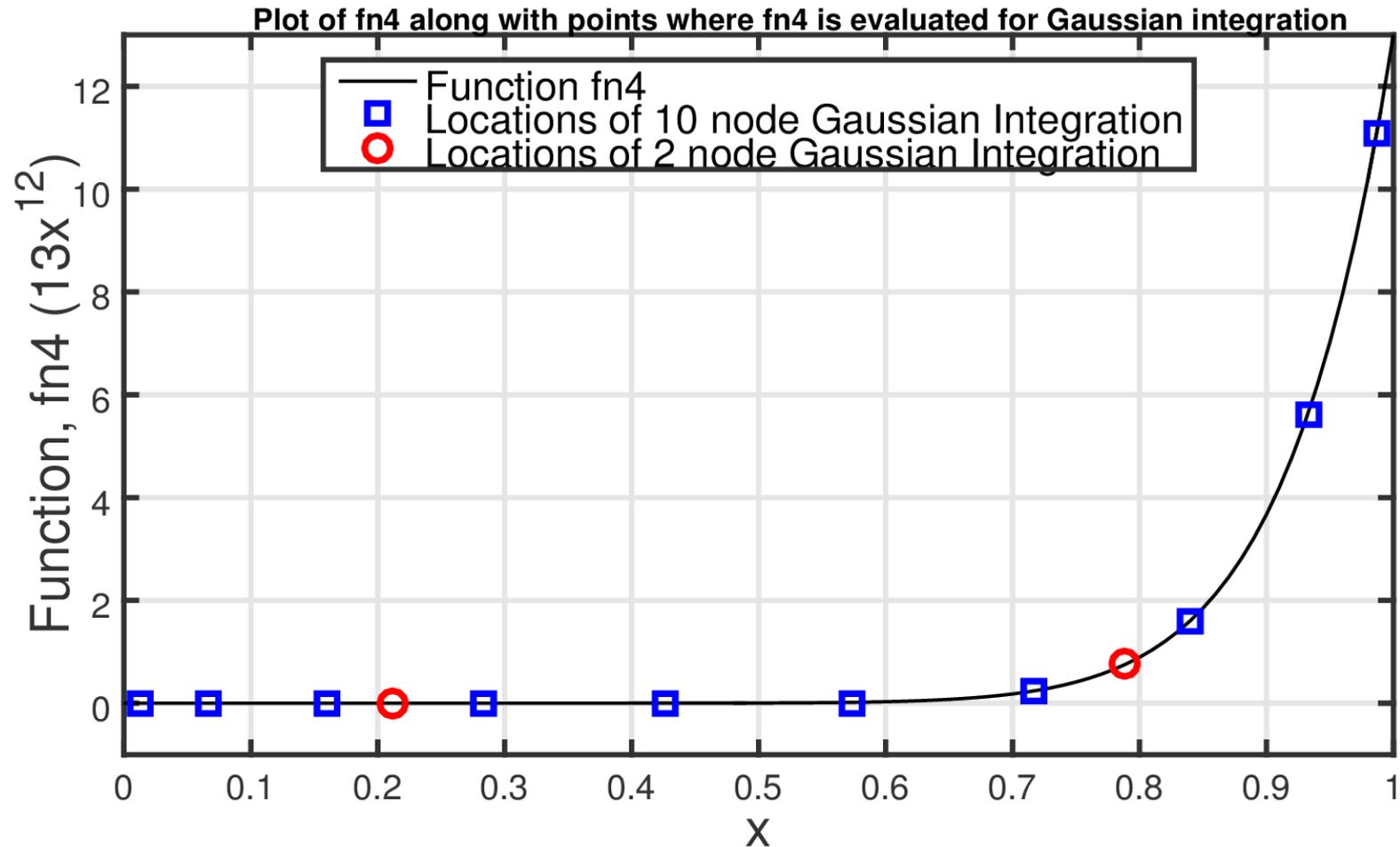
Gaussian Integration

Note: Gaussian quadrature acts as if we had fit a series of high order polynomials to the function, so a plot showing the nodes connected by straight lines does not represent how this method works



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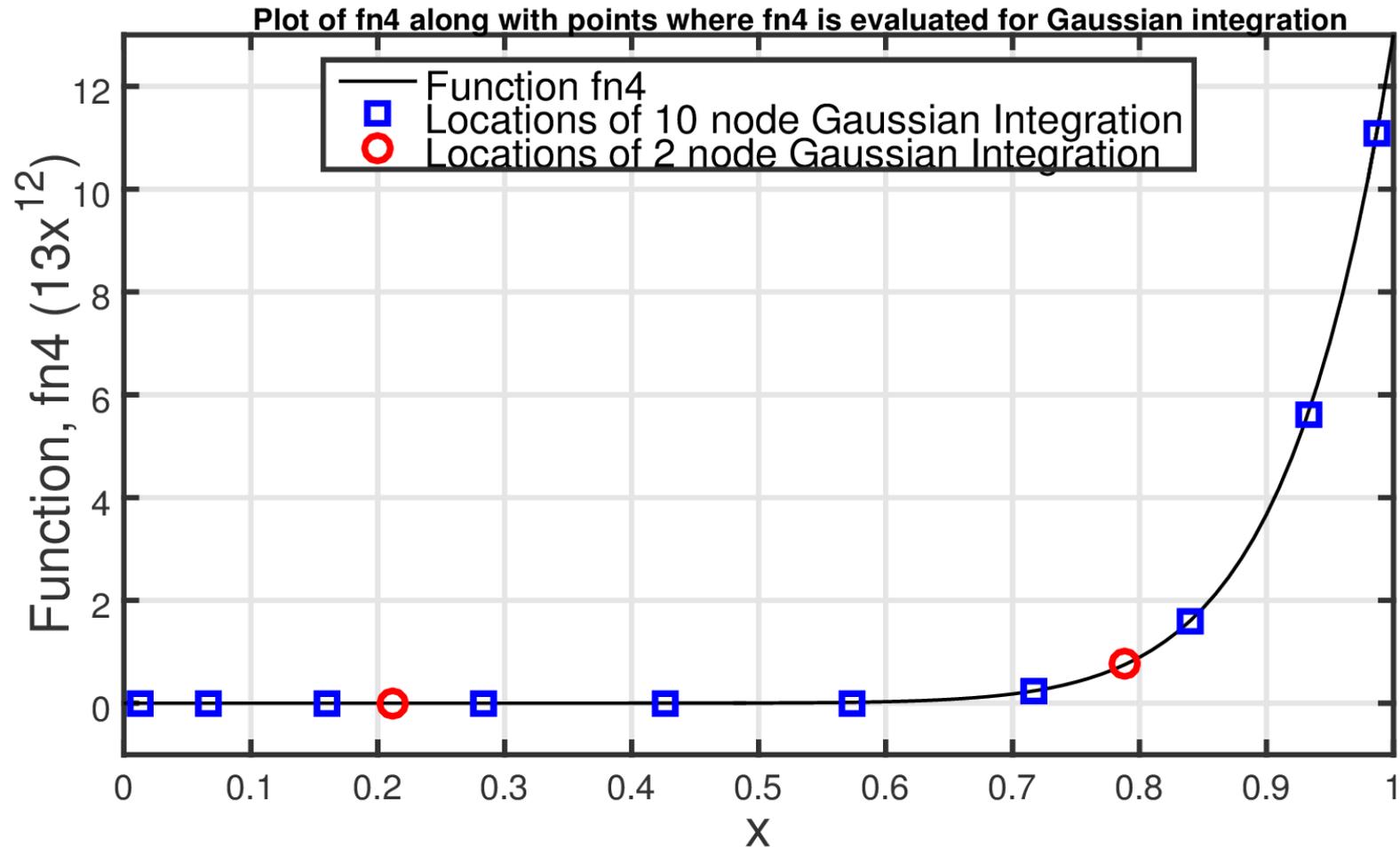
Gaussian Integration



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Gaussian Integration

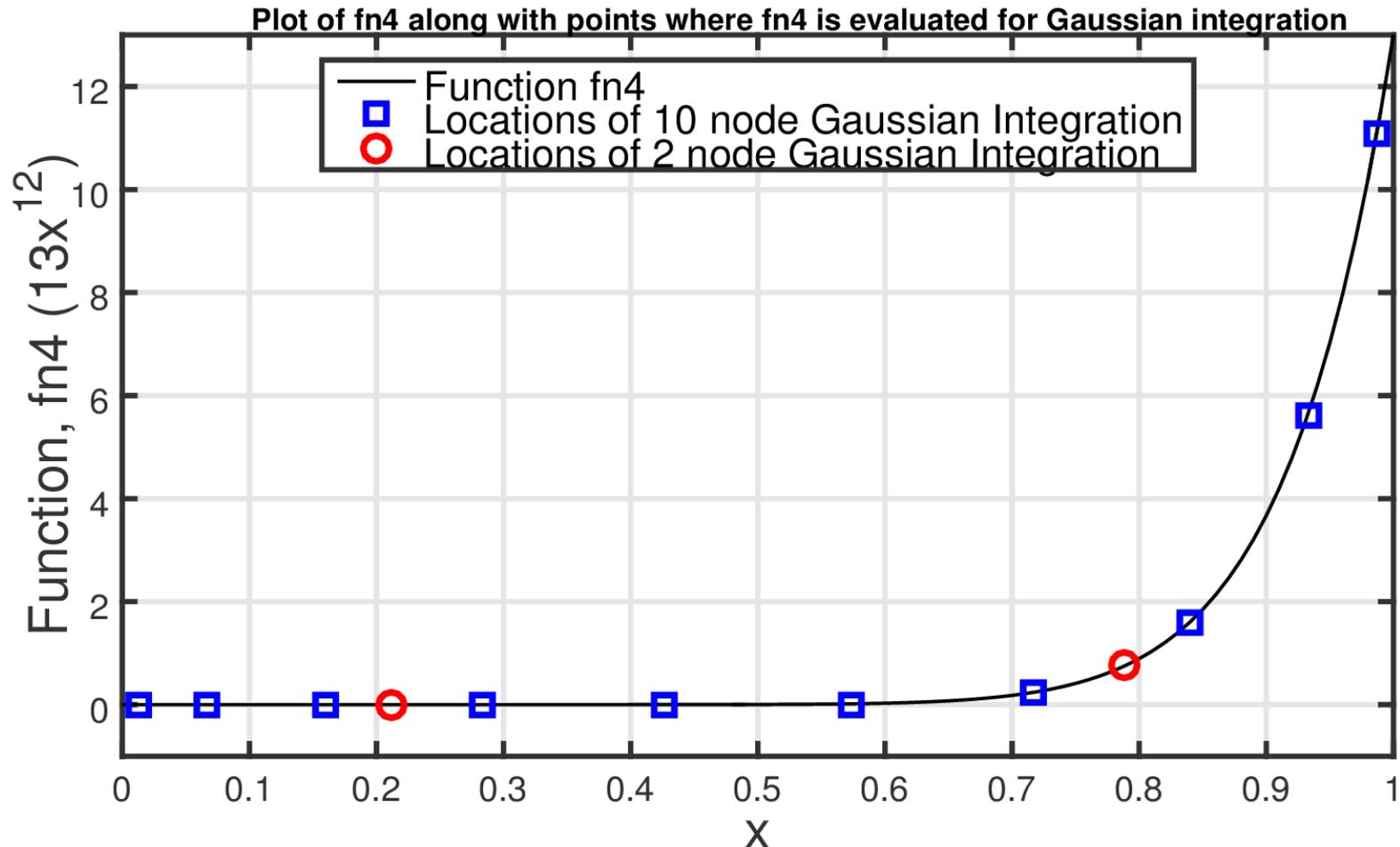
Less than full credit for part if your answer dealt in general statements such as “for 2 node approach, the assumed polynomials are only up to order 3, Which is not good enough” or “2 nodes doesn’t represent the whole plot”



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Gaussian Integration

Was looking for an answer with a specific reference to plot, such as “because of the location of the 2 nodes, the rapid rise of the function is not captured”; “the 10 nodes sample the function at enough locations to capture the rapid rise in value, near $X = 1$ ”



```

subroutine fn1(y,c)
  implicit double precision(a-h,o-z)
C
C
C Input : y [place where fn1(x) is to be evaluated]
C Output : c [value of fn1(x) for input value of y,
C           using an appropriate variable
C           transformation (see below) ]
C
C
C fn1(x) = 3x^2
C
C y is the variable used for Gaussian integration;x is the original variable
C The integral of the function is needed from x = 0. to 1.
C However, Gaussian integration by definition runs from y = -1. to 1.
C
C -----
C 1. a relation that describes x as a fn of y
C -----
C In Gaussian Quadrature, the x variable in f(x) will be
C substituted by  $g(y)=((b-a)/2)*y+(b+a)/2$ .
C
C -----
C 2. a relation that describes dx/dy
C -----
C dx/dy denotes the relationship between x and y, where is
C approximate to  $((b-a)/2)*ci$ . ci is the Gauss coefficient.
C
C The total equation of Gaussian Quadrature would be:
C
C  $((b-a)/2)*\text{Sigma}(ci*f(g(y)))=((b-a)/2)*\text{Sigma}(ci*f(((b-a)/2)*y+(b+a)/2))$ 
C
C In this function fn1, the calculaters:
C  $x= g(y) = ((b-a)/2)*y+((b+a)/2)$ 
C  $dx\_dy= ((b-a)/2)$ 
C output  $c=f(g(y))*dx\_dy$ 
C
C Note: because x integrate from 0 to 1,  $(b-a)/2=(b+a)/2=1/2$ 
C weight(i)=ci is given in main program.
C =====
C
C x=0.5*y+0.5
C dx_dy=0.5
C c=3.d0*(x**2)*dx_dy
C
C
C return
C end

```

```

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C =====
  x=0.5*y+0.5
  dx_dy=0.5
  c=3.d0*(x**2)*dx_dy

  return
end

```

Outstanding commenting!

**Comments are so very helpful,
to you and others, and good
commenting of code is vitally
important for group research efforts**

AOSC 652: HW 06, Part 1 Points Subtracted

No reference to quantitative errors from Simpson's rule and/or precision of Gaussian integration, in reply for which we would choose	Up to 3
Explanation of why 10-node works better than 2-node, couched in generalities rather than specifics	Up to 5
Attempt to explain Gaussian integration, as if operated like the Trapezoidal Rule	Up to 5
Problem with code used to determine where nodes should be placed	Up to 5
Problem with plot	Up to 10

AOSC 652: Analysis Methods in AOSC

US Standard Atmosphere

Geo potential Altitude above Sea Level - h - (m)	Temperature - t - ($^{\circ}C$)	Acceleration of Gravity - g - (m/s^2)	Absolute Pressure - p - ($10^4 N/m^2$)	Density - ρ - ($10^{-1} kg/m^3$)	Dynamic Viscosity - μ - ($10^{-5} N.s/m^2$)
-1000	21.50	9.810	11.39	13.47	1.821
0	15.00	9.807	10.13	12.25	1.789
1000	8.50	9.804	8.988	11.12	1.758
2000	2.00	9.801	7.950	10.07	1.726
3000	-4.49	9.797	7.012	9.093	1.694
4000	-10.98	9.794	6.166	8.194	1.661
5000	-17.47	9.791	5.405	7.364	1.628
6000	-23.96	9.788	4.722	6.601	1.595
7000	-30.45	9.785	4.111	5.900	1.561
8000	-36.94	9.782	3.565	5.258	1.527
9000	-43.42	9.779	3.080	4.671	1.493
10000	-49.90	9.776	2.650	4.135	1.458
15000	-56.50	9.761	1.211	1.948	1.422
20000	-56.50	9.745	0.5529	0.8891	1.422
25000	-51.60	9.730	0.2549	0.4008	1.448
30000	-46.64	9.715	0.1197	0.1841	1.475
40000	-22.80	9.684	0.0287	0.03996	1.601
50000	-2.5	9.654	0.007978	0.01027	1.704
60000	-26.13	9.624	0.002196	0.003097	1.584
70000	-53.57	9.594	0.00052	0.0008283	1.438
80000	-74.51	9.564	0.00011	0.0001846	1.321

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6000	-23.96	9.788	4.722	6.601	1.595
7000	-30.45	9.785	4.111	5.900	1.561

Almost certainly, T was specified as a function of altitude, and then pressure was found by integrating the hydrostatic equation.

30000	-48.04	9.715	0.1197	0.1641	1.475
40000	-22.80	9.684	0.0287	0.03996	1.601
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Almost certainly, T was specified as a function of altitude, and then pressure was found by integrating the hydrostatic equation.

What clue do we have that this is how the calculation was done ?

30000	-48.04	9.715	0.1197	0.1641	1.475
40000	-22.80	9.684	0.0287	0.03996	1.601
50000	-2.5	9.654	0.007978	0.01027	1.704
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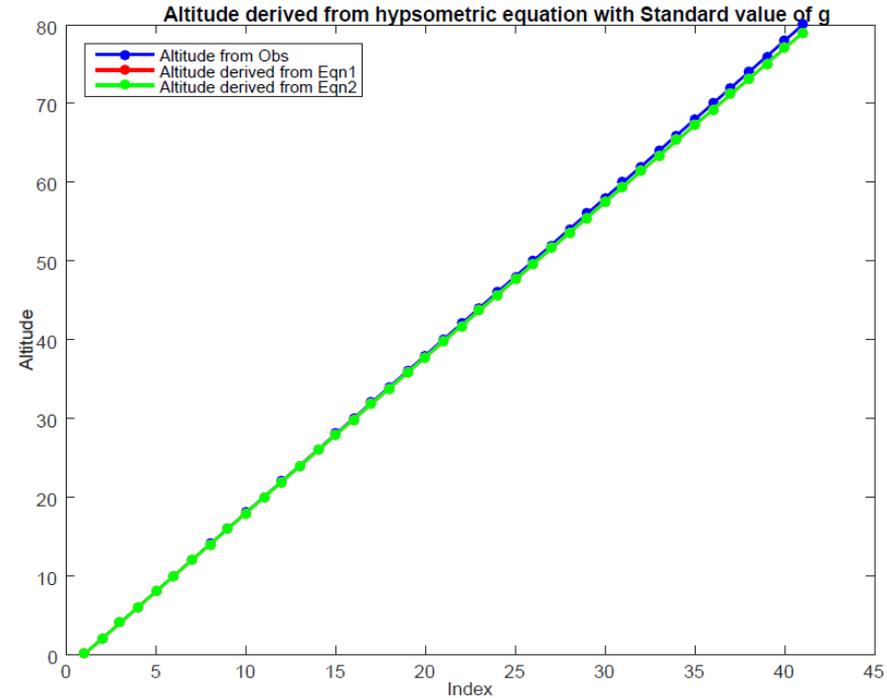
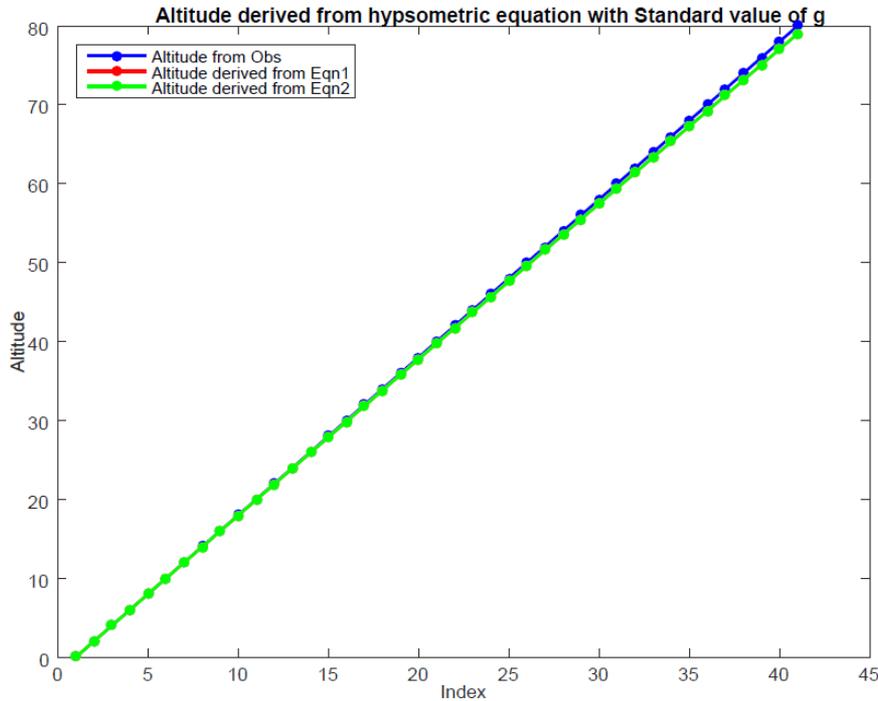
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So, our exercise was a round about way of seeing if we could “reverse” engineer altitude (i.e., solve for altitude as a function of p and T), even though we know this is not how the numbers in the US Std Atmosphere were generated.

As a result of carrying out this calculation, what have we learned ?

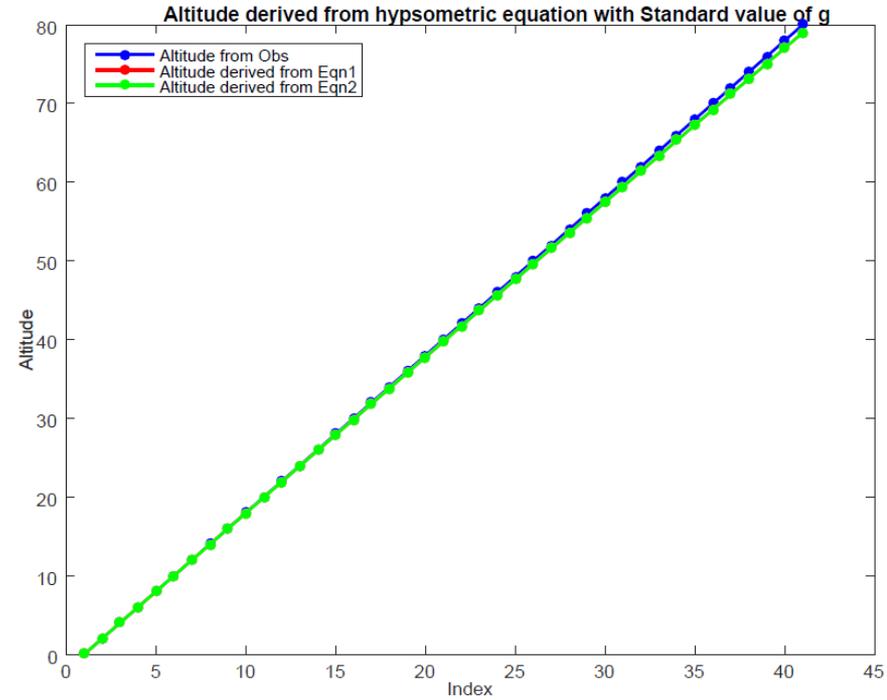
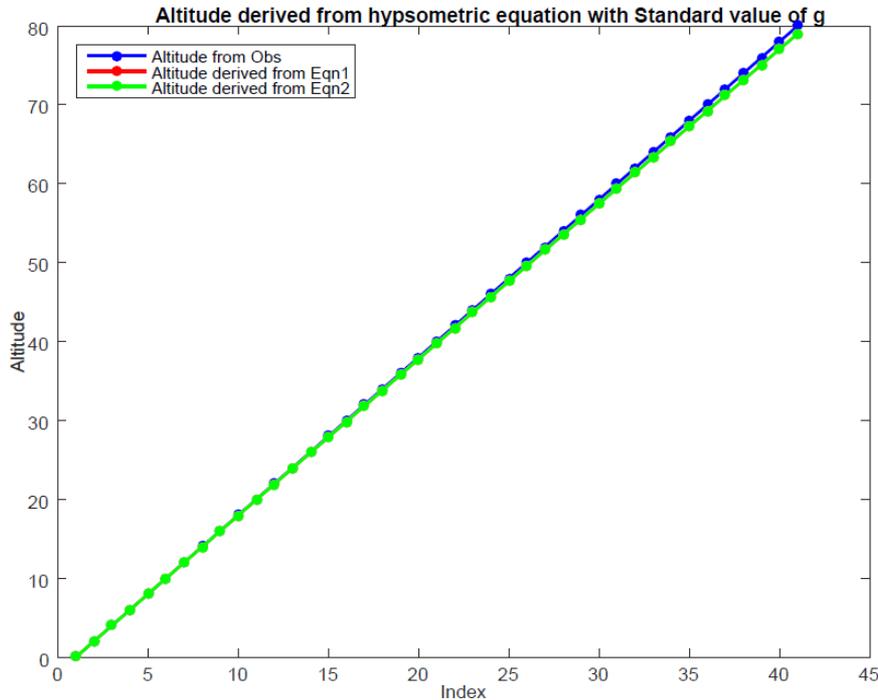
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The trapezoidal rule, despite “disdain” from numerical methods folks, works rather well (but not perfectly)

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If we want high precision, can not move “g” outside of the integral sign, as is often done in textbooks.

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9000	-43.43	9.779	3.099	4.694	1.493
10000	-49.92	9.776	2.693	4.180	1.459
11000	-56.41	9.773	2.350	3.721	1.425
12000	-62.90	9.770	2.068	3.315	1.391
13000	-69.39	9.767	1.844	2.959	1.357
14000	-75.88	9.764	1.675	2.650	1.323
15000	-82.37	9.761	1.558	2.386	1.289
16000	-88.86	9.758	1.491	2.164	1.255
17000	-95.35	9.755	1.471	1.980	1.221
18000	-101.84	9.752	1.495	1.831	1.187
19000	-108.33	9.749	1.561	1.715	1.153
20000	-114.82	9.746	1.666	1.630	1.119
21000	-121.31	9.743	1.808	1.573	1.085
22000	-127.80	9.740	1.985	1.542	1.051
23000	-134.29	9.737	2.196	1.534	1.017
24000	-140.78	9.734	2.440	1.546	0.983
25000	-147.27	9.731	2.716	1.576	0.949
26000	-153.76	9.728	3.023	1.621	0.915
27000	-160.25	9.725	3.360	1.680	0.881
28000	-166.74	9.722	3.727	1.751	0.847
29000	-173.23	9.719	4.123	1.833	0.813
30000	-179.72	9.716	4.548	1.925	0.779
31000	-186.21	9.713	5.001	2.027	0.745
32000	-192.70	9.710	5.481	2.138	0.711
33000	-199.19	9.707	6.000	2.257	0.677
34000	-205.68	9.704	6.557	2.383	0.643
35000	-212.17	9.701	7.152	2.515	0.609
36000	-218.66	9.698	7.785	2.652	0.575
37000	-225.15	9.695	8.456	2.793	0.541
38000	-231.64	9.692	9.165	2.938	0.507
39000	-238.13	9.689	9.911	3.086	0.473
40000	-244.62	9.686	10.693	3.237	0.439
41000	-251.11	9.683	11.511	3.390	0.405
42000	-257.60	9.680	12.364	3.545	0.371
43000	-264.09	9.677	13.251	3.701	0.337
44000	-270.58	9.674	14.172	3.858	0.303
45000	-277.07	9.671	15.127	4.015	0.269
46000	-283.56	9.668	16.115	4.171	0.235
47000	-290.05	9.665	17.136	4.326	0.201
48000	-296.54	9.662	18.189	4.480	0.167
49000	-303.03	9.659	19.273	4.633	0.133
50000	-309.52	9.656	20.388	4.784	0.099
51000	-316.01	9.653	21.533	4.933	0.065
52000	-322.50	9.650	22.707	5.080	0.031
53000	-329.00	9.647	23.909	5.224	0.000
54000	-335.49	9.644	25.139	5.365	0.000
55000	-341.98	9.641	26.396	5.503	0.000
56000	-348.47	9.638	27.679	5.638	0.000
57000	-354.96	9.635	28.987	5.770	0.000
58000	-361.45	9.632	30.319	5.899	0.000
59000	-367.94	9.629	31.674	6.025	0.000
60000	-374.43	9.626	33.051	6.148	0.000
61000	-380.92	9.623	34.449	6.268	0.000
62000	-387.41	9.620	35.867	6.385	0.000
63000	-393.90	9.617	37.295	6.499	0.000
64000	-400.39	9.614	38.732	6.610	0.000
65000	-406.88	9.611	40.178	6.718	0.000
66000	-413.37	9.608	41.633	6.823	0.000
67000	-419.86	9.605	43.096	6.925	0.000
68000	-426.35	9.602	44.567	7.024	0.000
69000	-432.84	9.599	46.045	7.120	0.000
70000	-439.33	9.596	47.529	7.213	0.000
71000	-445.82	9.593	49.019	7.303	0.000
72000	-452.31	9.590	50.514	7.390	0.000
73000	-458.80	9.587	52.014	7.474	0.000
74000	-465.29	9.584	53.518	7.555	0.000
75000	-471.78	9.581	55.026	7.633	0.000
76000	-478.27	9.578	56.538	7.708	0.000
77000	-484.76	9.575	58.053	7.780	0.000
78000	-491.25	9.572	59.571	7.849	0.000
79000	-497.74	9.569	61.092	7.915	0.000
80000	-504.23	9.566	62.615	7.978	0.000

Why could we not use Simpson's rule ?

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9000	-43.43	9.779	3.099	4.679	1.493
10000	-49.92	9.776	2.694	4.157	1.459
11000	-56.41	9.773	2.354	3.693	1.425
12000	-62.90	9.770	2.077	3.285	1.391
13000	-69.39	9.767	1.861	2.931	1.357
14000	-75.88	9.764	1.694	2.630	1.323
15000	-82.37	9.761	1.575	2.381	1.289
16000	-88.86	9.758	1.493	2.173	1.255
17000	-95.35	9.755	1.437	2.004	1.221
18000	-101.84	9.752	1.396	1.871	1.187
19000	-108.33	9.749	1.368	1.771	1.153
20000	-114.82	9.746	1.342	1.691	1.119
21000	-121.31	9.743	1.318	1.628	1.085
22000	-127.80	9.740	1.295	1.579	1.051
23000	-134.29	9.737	1.273	1.542	1.017
24000	-140.78	9.734	1.252	1.515	983
25000	-147.27	9.731	1.232	1.496	949
26000	-153.76	9.728	1.213	1.483	915
27000	-160.25	9.725	1.194	1.475	881
28000	-166.74	9.722	1.176	1.471	847
29000	-173.23	9.719	1.159	1.471	813
30000	-179.72	9.716	1.143	1.474	779
31000	-186.21	9.713	1.128	1.480	745
32000	-192.70	9.710	1.114	1.488	711
33000	-199.19	9.707	1.100	1.498	677
34000	-205.68	9.704	1.087	1.510	643
35000	-212.17	9.701	1.074	1.523	609
36000	-218.66	9.698	1.062	1.537	575
37000	-225.15	9.695	1.050	1.552	541
38000	-231.64	9.692	1.039	1.568	507
39000	-238.13	9.689	1.028	1.584	473
40000	-244.62	9.686	1.018	1.601	439
41000	-251.11	9.683	1.008	1.618	405
42000	-257.60	9.680	1.000	1.635	371
43000	-264.09	9.677	0.992	1.653	337
44000	-270.58	9.674	0.985	1.671	303
45000	-277.07	9.671	0.978	1.690	269
46000	-283.56	9.668	0.972	1.709	235
47000	-290.05	9.665	0.966	1.728	201
48000	-296.54	9.662	0.961	1.748	167
49000	-303.03	9.659	0.956	1.768	133
50000	-309.52	9.656	0.952	1.788	99
51000	-316.01	9.653	0.948	1.808	65
52000	-322.50	9.650	0.944	1.828	31
53000	-329.00	9.647	0.941	1.848	0
54000	-335.49	9.644	0.938	1.868	0
55000	-341.98	9.641	0.935	1.888	0
56000	-348.47	9.638	0.933	1.908	0
57000	-354.96	9.635	0.931	1.928	0
58000	-361.45	9.632	0.929	1.948	0
59000	-367.94	9.629	0.927	1.968	0
60000	-374.43	9.626	0.926	1.988	0
61000	-380.92	9.623	0.925	2.008	0
62000	-387.41	9.620	0.924	2.028	0
63000	-393.90	9.617	0.923	2.048	0
64000	-400.39	9.614	0.922	2.068	0
65000	-406.88	9.611	0.921	2.088	0
66000	-413.37	9.608	0.920	2.108	0
67000	-419.86	9.605	0.920	2.128	0
68000	-426.35	9.602	0.919	2.148	0
69000	-432.84	9.600	0.919	2.168	0
70000	-439.33	9.597	0.918	2.188	0
71000	-445.82	9.594	0.918	2.208	0
72000	-452.31	9.591	0.917	2.228	0
73000	-458.80	9.588	0.917	2.248	0
74000	-465.29	9.585	0.916	2.268	0
75000	-471.78	9.582	0.916	2.288	0
76000	-478.27	9.579	0.915	2.308	0
77000	-484.76	9.576	0.915	2.328	0
78000	-491.25	9.573	0.914	2.348	0
79000	-497.74	9.570	0.914	2.368	0
80000	-504.23	9.567	0.913	2.388	0

Why could we not use Simpson's rule ?

Because points were not evenly spaced with respect to the integrating variable, which in our case is $\ln(p)$

AOSC 652: Analysis Methods in AOSC

Numerical Integration

Simpson's Rule:

$$\int_{x_1}^{x_N} f(x) dx \approx \sum_{i=1, 3, 5}^{N-2} \frac{x_{i+2} - x_i}{2} \frac{f(x_{i+2}) + 4f(x_{i+1}) + f(x_i)}{3}$$

Classic method will work only for evenly spaced grid and odd N

Simpson's rule requires there be an odd number of points and that these odd number of points be evenly spaced!

AOSC 652: HW 06, Part 2 Points Subtracted

Not fixing my <i>intentional</i> mistake, regarding number of header lines	3
Not being specifically quantitative, when comparing results found using our integration to the expected value. Note: "slight difference" just doesn't cut it in the real world. 0.2 km over-estimate at 80 km much better	Up to 5
Not understanding that Simpson's Rule requires integrating interval to be evenly spaced	Up to 5
Coding error, calculation of zintegral	Up to 10

Output of original code

2,3

Index,Altitude

Estimate of altitude based on the evaluation of the integral of

R/grav Temperature d ln (pressure), for data read from file atm.us45std.dat

1	0.00
2	2.00
3	4.00
4	5.99
5	7.99
6	9.99
7	11.98
8	13.97

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Correct entry for number of header lines, either fixed by hand (a few students) or in Fortran (a few other students)

2,4

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3 4.00
4 5.99
5 7.99
6 9.99
7 11.98
8 13.97

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Simpson's rule requires there be an odd number of points and that these odd number of points be evenly spaced!

**How did we implement a version of Simpson's rule that works for either even or odd number of points ?
(they still must be evenly spaced!)**

AOSC 652: Analysis Methods in AOSC

Extra credit:

Traditionally, Simpson's rule can only be applied if the number of evaluation points is odd. How did we get around this restriction?

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We formed two branches.

Branch 1: if # of points is odd, use Simpson's rule

Branch 2: if # of points even, use Simpson's rule for $N-1$ of the intervals, and use Trapezoidal rule for remaining interval

AOSC 652: Analysis Methods in AOSC

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Why do we compare diff1 and diff2 ?

diff1 and diff2 represent the steepness of the slope at the respective ends of the function.

The trapezoidal rule is evaluated for the end interval with the smallest slope, so that the error in using the Trapezoidal rule will have the least impact.

AOSC 652: Analysis Methods in AOSC

Between now and Monday:

IDL Tract:

**Please read chapters 1, 2, and 3 of Bowman
*An Introduction to Programming with IDL***

Python Tract:

**Please read *, *, * of DeCaria,
*Python Programming and Visualization for Scientists***