Sub-mesoscale ocean vortex trains in the Luzon Strait

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This study uses the Argos satellite-tracked surface drifter trajectory data and ENVISAT (European satellite) ASAR (advanced synthetic aperture radar) images to illustrate the ocean vortex trains (OVT) in the Luzon Strait. Two cases that occurred in the northwest of Babuyan Island are observed. The first train of three cyclonic vortices showed up on drifter trajectories from 20° to 20.5°N and from 120° to 121°E, and the second, consisting of five pairs of cyclonic-anticyclic vortices, occurred on the upstream side of the first one from 19.5° to 20.0°N and from 121.0° to 122.0°E and showed up on the ASAR images acquired on 19 November 2006. The total length of the vortex train axis reaches about 250 km. All vortices propagate northwestward (≈315° TN). The mean angular velocity is \((2.07 \pm 0.18) \times 10^{-2} \text{s}^{-1}\). Theoretical models of ocean vortex radar image derived from radar imaging theories are used to extract dynamical parameters from ASAR imagery signatures, which include the distance between two consecutive vortices and that between two rows of vortices of (22.6 ± 1.9) km and (8.2 ± 1.2) km, respectively, the maximum rotational velocity radius as 4.70 km, and the vortex rotational angular velocity \(3 \times 10^{-5} \text{s}^{-1}\). Dynamical analyses give the mean velocity of the current of 0.65 ms\(^{-1}\), and the propagation velocity of the vortex 0.58 ms\(^{-1}\). The vortex shedding rate is estimated as 2.57 \(\times 10^{-5} \text{s}^{-1}\). The Reynolds number is estimated as 50 to 500. For the individual vortex and the vortex train, the Rossby numbers are \(O(0.4)\), and \(O(0.5)\), respectively, implying that both vortex and vortex train observed in the Luzon Strait have a sub-mesoscale nature. This study also reveals a strong current with an average surface current velocity of around 0.7 ms\(^{-1}\) and the direction of around 315° TN flowing directly from the Pacific to SCS passing through the southern Luzon Strait. The mean flow velocity can be calculated using methods developed in this study and OVT dynamical models. This information may provide more insight to the circulation systems in the area including the origin of Kuroshio.


1. Introduction

The vortex train is a phenomenon well documented in classical text books of fluid dynamics [e.g., Kundu, 1990]. Field observations and laboratory simulations indicate that when an object is placed in a steadily flowing fluid, the object will form a source of disturbance to the fluid flow. The downstream flow behind this obstacle will become unstable. Once the flow speed \(U\) and the body width \(D\) satisfy a certain relation (Reynolds number \(UD/\nu \gg 10\), in which \(\nu\) is the viscosity of fluid), two parallel rows of alternating vortices with opposite rotations will be generated. This kind of vortex train is commonly called a Kármán vortex street (KVS) [von Kármán, 1954; Chopra and Hubert, 1964; Taneda, 1965; Kundu, 1990].

In recent years, the KVS in the lower atmosphere occurring on the lee side of the separate mountain peaks or islands has been studied using the satellite images. On visible band images, the atmospheric KVS shows up through modulated cloud patterns [Tsuchiya, 1969; Thomson et al., 1977; Turner and Warren, 1988]. While on synthetic aperture radar (SAR) images, the atmospheric KVS shows up through modulation to the spatial distribution of ocean surface small waves [Li et al., 2000]. These satellite images can provide a two-dimensional, plane-view of KVS. By interpreting the satellite images, the fine structures and dynamic parameters of KVS can be obtained [Li et al., 2000].
This study deals with the vortex trains in the ocean. The vortex trains are observed in the Luzon Strait by the Argos satellite-tracked surface drifters and ENVISAT ASAR images acquired at the nearby Hong Kong Satellite Remote Sensing Ground Receiving Station. The significance of this study is not only to report the observed phenomenon, but also to develop methods for deriving of the dynamical parameters of OVT and mean flow. The next section gives a brief description of the geographic and hydrographic features of the study area. Section 3 contains satellite observations of the phenomenon and air-sea boundary conditions. Section 4 derives theoretical models of vortex radar image. On the basis of the models and radar backscatter section data taken from the ASAR images, the dynamical parameters of the vortex and vortex train are measured or calculated. Section 4 analyzes dynamical features of the OVT observed in the Luzon Strait. Sections 5 and Sections 6 give discussion and conclusions, respectively.

2. Study Area

2.1. Brief Geography

The Luzon Strait, a study area of this paper, lies between Taiwan Island and Luzon Island as shown in Figure 1. The strait is approximately 386 km wide from south to north. The average depth is 1400 m [Guo et al., 2004]. The strait connects the South China Sea (SCS) on the west and the Philippine Sea on the east. On the east side of the strait, there are two groups of islands: the Batan Islands in the north and Babuyan Islands in the south. These two groups of islands divide the strait into three smaller channels: the Bashi Channel, the northernmost one, which separates Taiwan from the Batan Islands, the Balintang Channel is in the middle, which separates the two islands, and the Babuyan Channel is the southernmost one, which separates Luzon Island from the Babuyan Islands.

2.2. Satellite-Track Drifter Trajectory Data

During the period from 1979 to 2003, a great number of Argos satellite-tracked surface drifters were released in the Kuroshio and adjacent waters by the Tropical Ocean Global Atmosphere (TOGA) program, the World Ocean Circulation Experiment-Surface Velocity Program (WOCE-SVP), and Chinese oceanographic research programs. Figure 1 shows trajectories in and near the study area in November only. The drifter drogue was placed at a water depth of 15 m, so that the direct effects of atmospheric processes through sea surface winds on the drifter movement and current velocity calculation were ignorable [Niler et al., 1995]. The temporal resolution of drifter positioning for the data used in this study was one day [Hansen and Poulain, 1996].

From Figure 1, one can see that there are six trajectories passing through the southern Luzon Strait. In particular, the two trajectories passing through Babuyan Islands show a train of three cyclonic vortices distributed from 20° to 20.5°N and from 120° to 121°E, and another one just behind Camiguin Island. The average distance between two centers of consecutive cyclonic vortices in the train is 50 km. The vortices occurred in the wake of Babuyan Islands, which serves as blunt obstacles in the ocean flow, and formed a vortex train in a line behind the islands. These striking features evidence their nature of OVT. The trajectories show that all the drifters moved northwestward (~315° TN), implying that a northward current flows from the Pacific into the SCS. The daily average current velocities derived from the trajectories are 1.3 m s⁻¹ at the east border of the Strait (19° to 20.5°N, 121.5°E) and 0.6 m s⁻¹ at the locations of vortex train (20° to 20.5°N, 120° to 121°E), respectively. The same features once again show up on the ENVISAT ASAR images of 19 November 2006 as shown in Figures 3a and Figures 3b and will be analyzed in details.

2.3. Estimated Parameters of the Vortex Train

An enlarged map showing two Argos drifter trajectories with the vortex train signal is shown in Figure 2. The dynamical parameters of the vortices in the vortex train are measured for subsequent analysis. A period of drifter rotating around a vortex, T_d, can be determined directly from the trajectories. Thus the rotational angular velocity of
the vortex, $\omega_d$, can be estimated by $2\pi/T_d$. A vortex radius at which the drifter rotated, $R$, is determined by half of the maximum distance of vortex trajectory outedge. The radius determined by this method is an average distance of drifter to the vortex center, which does not necessarily coincide with the radius of the maximum rotational velocity. Then the drifter-determined daily mean tangent velocity can be estimated by $\omega_d R$. The results are listed in Table 1, in which the vortices are coded from number 1 in a northwestward order. One can see that the mean angular velocity of the vortices reaches $(2.66 \pm 0.56) \times 10^{-5} \text{s}^{-1}$, 4–5 times faster than the Gulf Stream meso-scale eddies [Zheng et al., 1984; Zheng and Yuan, 1989].

3. Satellite Observations and Air-Sea Boundary Conditions

3.1. ENVISAT ASAR Images

The previous results have indicated that satellite SAR is a powerful sensor to record ocean processes. Its all weather and all time capacities break through restrictions of cloud cover and darkness, and its imaging resolution, up to tens of meters, enables us to detect the fine structure of the ocean processes. In particular, SAR instruments are highly sensitive to the variation of spatial distribution of the sea surface roughness, which results from modulation effects of dynamical processes in the lower atmosphere and ocean [Fu and Holt, 1982; Meng, 2002; Zheng, 2005; Zheng et al., 2001, 2004, 2006, 2007].

The satellite SAR images used in this study are taken by ENVISAT, which was launched into a sun synchronous orbit by European Space Agency on 1 March 2002. Its orbit height is 786 km and inclination 98°. One of the most important payloads onboard ENVISAT is a C-band (5.331 GHz) SAR, which uses an active phased-array antenna with incidence angles between 15 and 45 degrees to provide dual polarization, and wide swath coverage imaging data. In Alternating Polarization Mode (AP Mode), the antenna provides three choices for different transmitting and receiving polarization combinations: VV and HH, or HH and HV, or VV and VH.

The ASAR images used in this study were acquired and processed at the Hong Kong Satellite Remote Sensing.
Ground Receiving Station, which was established at the Chinese University of Hong Kong in 2006. The images are generated by ASAR Image Mode. The nominal spatial resolution is 30 m, and the swath coverage is 100 km. The receivable area of the antenna of the Hong Kong Station covers a circular region of 2500 km in radius centered at Hong Kong, enclosing most of the East Asia continent, Northwest Pacific from the Japan Sea to the Java Sea, and the Bay of Bengal in the North Indian Ocean.

The study area is 600 to 800 km from Hong Kong Station. Thus high quality images, low noise, and low distortion may be ensured. Figure 3a shows a mosaic of ASAR images acquired at 13:53:45 UTC on 19 November 2006. The image coverage is shown in an inset on top right corner. A vortex train shows up in the dashed line box.

**Figure 3a.** A mosaic of ASAR images acquired at 13:53:45 UTC on 19 November 2006. The image coverage is shown in an inset on top right corner. A vortex train shows up in the dashed line box.
the northwest of Babuyan Island. An enlarged image of this
sub-region is shown in Figure 3b. One can see that the
vortex train imagery consists of five pairs of dark-bright,
_near-circular, vortex-like patterns, and the Babuyan Island
looks like a train head.

3.2. Weather Conditions for ASAR Imaging

[15] The surface analysis weather charts before
(12:00 UTC) and after (18:00 UTC) ASAR imaging are
shown in Figures 4a and 4b. One can see that over adjacent
East China, a weak high pressure (1024 mb) was moving
eastward, meanwhile a cold front east of Taiwan was
moving southeastward. While these two features had not
yet influenced the study area at the imaging time. Over the
study area were a clear sky and low wind conditions (wind
speed <5 ms⁻¹). Weather conditions observed at the nearest
weather station, Itbayat Island located at 20°47'N 121°51'E,
80 km north of the vortex train, are listed in Table 2.
[16] A MODIS (Moderate Resolution Imaging Spectroradiometer) thermal infrared composite image that was near-
simultaneously taken with ASAR image (51 min later) is
shown in Figure 5. One can see again the clear sky over the
study area and no strong weather systems showing up.

[17] The weather maps and satellite images all show a clear
and calm weather condition at the time of ASAR imaging.
The sea surface wind speed obtained by linear interpolation
was 2 ms⁻¹, implying a favorable condition for imaging of
the sea surface process. Meanwhile, these boundary condi-
tions further exclude the possibility of involvement of
atmospheric processes in the vortex train images.

4. ASAR Image Interpretation

4.1. Recognition of OVT

[18] In section 2.2, the OVT in the Luzon Strait has been
recognized from the Argos satellite-tracked drifter trajecto-
ries. In Figure 6, these trajectories are overlaid on the ASAR
image. One can see that the cyclonic vortices on the trajecto-
ries are just distributed in the downstream side of vortex
train on the ASAR image. The two vortex train axes consti-
tute a continuous straight line starting from the Babuyan
Island. This evidences the common nature of two vortex
trains, although they were not simultaneously observed.

[19] From the ASAR images shown in Figures 3a and 3b,
the following conclusion is immediately derived. (1) The
observation time was 13:53:45 UTC on 19 November 2006.
(2) The OVT occurred within a range of 50 by 100 km from
19.5° to 20.0°N and from 121.0° to 122.0°E in the Luzon
Strait. (3) The OVT consists of five pairs of cyclonic-
anticyclonic vortices that are distributed in the wake of
Babuyan Island. The axis of the OVT extends northwest-
ward (~315°) starting from the Babuyan Island, implying
existence of a northwestward mean flow-passing through
the Luzon Strait. Later it will be verified that the center of
the dark or bright area is the center of vortex, so that the
average distance between the two centers of consecutive
vortices and that between the two centers of adjacent
cyclonic-anticyclonic vortices are measured as listed in
Table 3.

4.2. Theoretical Models of Ocean Vortex Radar Image

[20] In order to extract dynamical parameters of the ocean
vortex from the ASAR image, theoretical models of ocean
vortex radar image must be derived from general radar
imaging theories for ocean processes. In other words, the
transformational relations between the radar imagery and
vortex truth must first be set up.

[21] In general, an ocean surface radar image records a
horizontal two-dimensional (Cartesian x-y) distribution of
radar return signal intensity. The latter is parameterized by
the backscatter cross section per unit area, which is defined
as [Plant, 1990]

\[ \sigma_r(\theta_0) = 16\pi k_0^4 |g_0(\theta_0)|^2 \psi(0, 2k_0 \sin \theta_0), \tag{1} \]

where \( \theta_0 \) is the incidence angle, \( k_0 \) is the wave number
of the radar waves, and \( \Psi \) is the two-dimensional wave
number spectral density of the ocean surface wavefield which
satisfies the Bragg resonant scatter condition, the incident
radiation is in the \( x-z \) plane (\( z \) being the vertical direction),
the indices \( ij \) denote the polarizations of the incident
and backscattered radiation, respectively, and \( g_0(\theta_0) \) are the
first-order scattering coefficients. For a given radar wave
number and an incidence angle, the radar return signal intensity
depends only on the two-dimensional wave number spectral
density of the ocean surface wavefield, \( \Psi \). The spatial
variability of \( \Psi \) results in the intensity variability on radar
images, so that generates imagery patterns or signatures of
ocean processes.

<table>
<thead>
<tr>
<th>Vortex Time (yy/mm/dd)</th>
<th>Period of Drifter, Day</th>
<th>Angular Velocity, ( 10^{-3} ) s⁻¹</th>
<th>Radius, km</th>
<th>Tangent Velocity, Day, ms⁻¹</th>
</tr>
</thead>
</table>
| 1 92/11/16 – 19 8.97 3.24 8.97 0.29
| 2 92/11/18 – 22 12.56 1.90 12.56 0.24
| 3 92/11/21 – 24 6.28 2.83 6.28 0.18
| Mean 2.66 ± 0.56 0.24 ± 0.04
Considering the modulation of small ocean surface waves by wave-current interaction, Yuan [1997] derived the wave number spectral density of the high frequency ocean surface wavefield in the form of

\[
Y = \frac{m_1}{c_0} \frac{3}{2} \frac{m_3}{c_3} \frac{c_1}{c_0} \frac{c_16}{c_17} \frac{c_2}{c_0} \frac{g}{c_1} \frac{k_2}{c_0} \frac{w}{c_0} \frac{S_{ab}}{U_b} \frac{\partial U_b}{\partial x} \frac{\partial w}{\partial c_18/c_19} \cos^2 \phi \cos \sin \phi + \frac{\partial v}{\partial y} \sin^2 \phi / 2.
\]

for gravity-capillary wave band (2)

\[
Y = \frac{m_1}{c_0} \frac{3}{4} \frac{m_3}{c_3} \frac{c_1}{c_0} \frac{c_16}{c_17} \frac{c_2}{c_0} \frac{g}{c_1} \frac{k_2}{c_0} \frac{w}{c_0} \frac{S_{ab}}{U_b} \frac{\partial U_b}{\partial x} \frac{\partial w}{\partial c_18/c_19} \cos \sin^2 \phi / 2.
\]

for capillary wave band (3)

where \( m_1, m_3, \) and \( m_4 \) are coefficients (\( m_3 \) is called the spectral constant in this study later on), \( c_0 \) is the friction velocity, \( c_3 \) is the wave phase speed, \( \gamma \) is the viscosity of seawater, \( \omega \) is the angular frequency of the ocean surface waves, \( k \) is the wave number of ocean surface waves, and \( S_{\alpha \beta} \) is the wave number spectral density of the wave-current interaction. From equations (1)–(4), one can see that if the wind field and seawater viscosity are homogeneous, the current velocity field is a determining factor to generate the radar image of an ocean process. In our case, the ocean vortex and coordinate system are shown as a schema in Figure 7. The origin of the 2D horizontal coordinate system is chosen as the vortex center. The \( x \)-axis is chosen as the ocean wave propagation direction, i.e., the sea surface wind direction. The \( y \)-direction is perpendicular to the \( x \)-axis. Assume that the vortex is a cyclonic eddy with a counterclockwise rotating angle \( \theta \) and a radial axis \( r \).

The rotational velocity structure of the vortex can be written as an empirical model [Zheng et al., 1984]

\[
V_0 = \frac{\omega \rho r^2}{r_0} \exp \left[ \frac{1}{2} - \frac{1}{2} \left( \frac{r}{r_0} \right)^4 \right],
\]

where \( \omega \) is the rotational angular velocity, and \( r_0 \) is a radius at which the velocity takes a maximum value. Figure 8 shows a graph of (5). The data points are taken from the drifter trajectories shown in Figure 1 as listed in Table 1 and normalized by the maximum velocity. One can see that the empirical model fits the data quite well.
On the other hand, the coordinate system setup implies that the small wave direction $\phi$ in equation (4) is zero. Thus (4) is simplified as

$$S_{ab} \frac{\partial U_b}{\partial x_a} = \frac{\partial u}{\partial x} = \frac{\partial V_\theta}{\partial r} \sin 2\theta - \frac{\partial V_r}{\partial r} \cos 2\theta.$$  \hspace{1cm} (6)

where $u = -V_\theta \sin \theta + V_r \cos \theta$, here $V_r \ll V_\theta$, so that the second term is negligible. Thus

$$\frac{\partial u}{\partial x} = \omega_\psi \sin 2\theta \left[ \frac{V_\theta}{r} - \frac{\partial V_\theta}{\partial r} \right].$$  \hspace{1cm} (7)

Substituting (5) into (7) yields

$$\frac{\partial u}{\partial x} = \omega_\psi \sin 2\theta \left[ 2 \left( \frac{r}{r_0} \right)^5 \frac{r}{r_0} \exp \left( \frac{1}{2} - \frac{1}{2} \left( \frac{r}{r_0} \right)^4 \right) \right].$$  \hspace{1cm} (8)

Assume that both wind field and seawater viscosity are homogeneous, $\Psi$ in equation (2) can be written as

$$\Psi = \Psi_0 + \Psi_e,$$  \hspace{1cm} (9)

where $\Psi_0$ represents a mean, homogeneous wavefield, and $\Psi_e$ represents a variability of wavefield induced by an ocean vortex.

$$\Psi_e = m_j^{-1} k^{-4} \omega^{-1} \omega_\psi \sin 2\theta \left[ \frac{r}{r_0} - 2 \left( \frac{r}{r_0} \right)^5 \right] \exp \left( \frac{1}{2} - \frac{1}{2} \left( \frac{r}{r_0} \right)^4 \right).$$  \hspace{1cm} (10)

Substituting (6) and (8) into (10) yields

$$\Psi_e = m_j^{-1} k^{-4} \omega^{-1} \omega_\psi \sin 2\theta \left[ \frac{r}{r_0} - 2 \left( \frac{r}{r_0} \right)^5 \right] \exp \left( \frac{1}{2} - \frac{1}{2} \left( \frac{r}{r_0} \right)^4 \right)/4.$$  \hspace{1cm} (11)

Substituting (11) into (1) yields the variability of backscatter cross section per unit area

$$\sigma_{0e} = 4\pi k_0^4 |g_0(\theta_0)|^2 m_j^{-1} k^{-4} \omega^{-1} \omega_\psi \sin 2\theta \left[ \frac{r}{r_0} - 2 \left( \frac{r}{r_0} \right)^5 \right] \exp \left( \frac{1}{2} - \frac{1}{2} \left( \frac{r}{r_0} \right)^4 \right).$$  \hspace{1cm} (12)

Table 2. Weather Conditions Observed Within 2 h of ASAR Imaging at Itbayat Island

<table>
<thead>
<tr>
<th>Time</th>
<th>Air Temp., $^\circ$C</th>
<th>Wind Speed, ms$^{-1}$</th>
<th>Wind Direction</th>
<th>Cloud Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>12:00Z</td>
<td>26</td>
<td>5</td>
<td>SE</td>
<td>25%</td>
</tr>
<tr>
<td>15:00Z</td>
<td>28</td>
<td>1</td>
<td>SE</td>
<td>25%</td>
</tr>
<tr>
<td>13:54Z$^a$</td>
<td>27$^a$</td>
<td>2$^a$</td>
<td>SE$^a$</td>
<td>25%$^a$</td>
</tr>
</tbody>
</table>

$^a$Imaging time and interpolated values.
Equation (12) is the theoretical model of an ocean cyclonic vortex radar image being looked for. For the rotational angular velocity $\omega_e$ takes a negative value, model (12) represents an anti-cyclonic vortex.

Figure 9 shows a comparison of the theoretical model with a $s_0e_0$ profile taken from a transect across the third cyclonic vortex behind Babuyan Island in Figures 3a and 3b. One can see that as a first order approximation, the model fits a general trend in radar imagery, in particular, the increasing edge. Obviously, the imagery also contains other high frequency signals or noises, which make the imagery deviate from the vortex model.

4.3. Inferences of Theoretical Models

4.3.1. The Center of True Vortex
Equation (12) indicates that the radar backscatter cross section $s_{0e}$ increases with $r$, for $r < r_0$. Let $r = 0$, (12) gives $s_{0e} = 0$. This means that the geometric center of the vortex imagery is the center of the true vortex. Thus the distances between two consecutive vortices and two rows of vortices can be measured from the ASAR image. These data are important parameters for dynamical analysis of OVT (see section 5). The measured results are listed in Table 3.

4.3.2. The Maximum Rotational Velocity Radius
Equations (5) and (12) show that the maximum rotational velocity radius, $r_0$, is a key parameter in determining the velocity structure of vortex. However, this parameter cannot directly be measured from the vortex radar image. A related parameter, which can be measured...
from radar image, is a maximum radar backscatter radius defined as $r_m$. According to the definition, $r_m$ satisfies

$$\left. \frac{\partial \sigma_0}{\partial r} \right|_{r=r_m} = 0. \quad (13)$$

Solving equation (13) results to the physically sound solutions

$$r_m = 0.542 r_0, \text{ with } \left. \frac{\partial^2 \sigma_0}{\partial r^2} \right|_{r=r_m} > 0, \quad (14a)$$

Table 3. Distances Between Two Consecutive Vortices and Two Rows of Vortices Measured From the ASAR Image

<table>
<thead>
<tr>
<th>Vortex Pair</th>
<th>Distance of Two Vortices (a) km</th>
<th>Distance of Two Vortex Row (h) km</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>6.30</td>
</tr>
<tr>
<td>3</td>
<td>22.92</td>
<td>8.60</td>
</tr>
<tr>
<td>4</td>
<td>22.92</td>
<td>8.02</td>
</tr>
<tr>
<td>5</td>
<td>22.92</td>
<td>9.74</td>
</tr>
<tr>
<td>Mean</td>
<td>22.6 ± 1.9</td>
<td>8.2 ± 1.2</td>
</tr>
</tbody>
</table>

Figure 6. A composite image of ASAR image with Argos satellite-tracked drifter trajectories of November.

Figure 7. A schema of the cyclonic vortex and the coordinate system.
Comparing with the theoretical curve in Figure 9, one can see that these two solutions correspond to the minimum value at pixel 34 and the maximum at pixel 82, respectively. Thus after \( r_m \) is measured from the radar image, the maximum rotational velocity vortex radius, \( r_0 \), can be calculated by (14a) or (14b). In the case shown in Figure 9, the maximum rotational velocity radius \( r_0 \) is determined to be 4.70 km.

4.3.3. The Vortex Rotational Angular Velocity and Spectral Constant \( m_3 \)

If the radar backscatter cross section \( \sigma_{0e} \) at a given \( r \) is obtained from an ocean vortex imagery, the rotational angular velocity \( \omega \) and spectral constant \( m_3 \) can be calculated by (12), if one of them is known. Taking the case of Figure 9 as an example, from radar imagery transect data we obtain the radar backscatter cross section at vortex center \( \sigma_{0e} (0) = 0.0149 \), which serves as a background backscatter cross section, and the maximum radar backscatter section at \( r = 6.15 \) km \( \sigma_{0e} (6.15 \) km) = 0.0276. Thus vortex-induced increment in radar backscatter cross section at \( r = 6.15 \) km \( \Delta \sigma_{0e} (6.15 \) km) should be 0.0127. Other parameters are as follows: \( \lambda_0 = 5.7 \) cm; \( \theta_0 = 30^\circ \); \( |g_0(\theta_0)|^2 = 0.929 \) (salinity 35 psu, water temperature 22°C, HH polarization); \( \omega = 35.26 \) s\(^{-1} \); \( \theta = -45^\circ \); \( r_o = 4.70 \) km, and

\[
\left[ \frac{r}{r_0} - 2 \left( \frac{r}{r_0} \right)^5 \right] \exp \left[ \frac{1}{2} - \frac{1}{2} \left( \frac{r}{r_0} \right)^4 \right]_{r=6.15 \text{ km}} = -2.42.
\]

Substituting these parameters into (12) yields

\[
\omega_E \exp^{-1} = 1.54 \times 10^{-2}.
\]

[38] On the other hand, the data listed in Table 1 indicate that a reasonable estimate of the vortex angular velocities should be \( \omega = 3 \times 10^{-5} \) s\(^{-1} \). Thus from (15) we obtain an estimate of spectral constant \( m_3 \) as 1.95 \times 10^{-3}.

4.4. Dynamical Analysis of OVT

A physical model of OVT is schematically shown in Figure 10. The parameters are defined as follows: \( U_0 \), the current velocity of mean or undisturbed flow; \( U_e \), the propagation velocity of vortex; \( a \), the distance between two consecutive vortices; and \( h \), the distance between two rows of vortices.

[40] Based on this model, the vortex propagation problem can be solved by solving the following two equations. The first equation describes the relation between \( U_0 \) and \( U_e \) [Chopra and Hubert, 1964]

\[
U_0 - U_e = \frac{K}{2a} \tanh\left( \frac{\pi h}{a} \right)
\]

(16)
where $K$ is defined as the vortex circulation strength

$$k = 2\pi r_0 V_0. \quad (17)$$

The second equation is derived by Tsuchiya [1969]

$$(2B - A)x^2 + (2A - 3B)\chi + (B - A + B/4A) = 0, \quad (18)$$

where $\chi = U_e/U_0$, $A = \coth (\pi h/a)$ and $B = \pi h/a$. In the case of interest, $h/a = 0.363$, so that $A = 1.229$, and $B = 1.139$. Substituting these parameters into (19) and taking a larger solution yields [Li et al., 2000]

$$\chi = U_e/U_0 = 0.885. \quad (19)$$

The reasonable estimate of angular velocity should be $3 \times 10^{-5}$ s$^{-1}$. The maximum velocity radius has been determined as $r_0 = 4.70$ km. Substituting these parameters into (18) yields the vortex circulation strength $K = 4161$ m$^2$s$^{-1}$. Substituting the $K$ value into (17) yields

$$U_0 - U_e = 0.0749 \text{ ms}^{-1}. \quad (20)$$

Solving (19) and (20) yields $U_0 = 0.65$ ms$^{-1}$ and $U_e = 0.58$ ms$^{-1}$. On the other hand, the drifter trajectories in November shown in Figure 1 give an average mean flow velocity $(0.70 \pm 0.25)$ ms$^{-1}$, which is coincided with the above estimates. Meanwhile, an estimate of the vortex shedding rate $F (= U_e/A)$ is derived as $2.57 \times 10^{-5}$ s$^{-1}$.

The Reynolds number is defined as [Kundu, 1990]

$$R_e = \frac{UD}{A_H}, \quad (21)$$

where $U = O(50 \text{ cms}^{-1})$ and $D = O(1 \times 10^6 \text{ cm})$ are the characteristic velocity of mean flow and length scale of island diameter, respectively, and $A_H$ is the horizontal turbulent viscosity. Thus in the case of interest a reasonable estimate of the Reynolds number is $R_e = O(50$ to 500), if $A_H$ is generally chosen as $1 \times 10^3 \text{ cm}^2$s$^{-1}$ to $1 \times 10^6 \text{ cm}^2$s$^{-1}$ for the mesoscale or sub-mesoscale ocean processes [Zheng and Yuan, 1989].

The Rossby number defined as

$$\varepsilon = \frac{U}{fL}, \quad (22)$$

where $f (=2\Omega \sin \Theta)$ is the local Coriolis parameter and $L$ is the length scale of phenomenon, is used to estimate the dynamical scale of a geophysical fluid phenomenon. From the Argos satellite-tracked drifter trajectory data, the mean angular velocity of vortices is determined as $(2.66 \pm 0.56) \times 10^{-5}$ s$^{-1}$. In the above section, the vortex shedding rate is estimated as $2.57 \times 10^{-5}$ s$^{-1}$ from ASAR images. While the local Coriolis parameter $f(\Theta = 20^\circ)$ is $4.97 \times 10^{-5}$ s$^{-1}$. Thus for an individual vortex, the Rossby number $\varepsilon$ is $O(0.4)$, and for the KVS, it is $O(0.5)$. This implies that vortex train observed in the study area has a dynamically sub-mesoscale nature, i.e., the length scale is $O(50 \text{ km})$, and the Rossby number is within a range of $0.1 < \varepsilon < 1$.

5. Discussion

There are similarities between the atmospheric KVS and oceanic vortex train. However, their spatial scales are distinct. For example, the distance between two consecutive atmospheric vortices usually is $O(100 \text{ km})$ [Tsuchiya, 1969; Thomson et al., 1977; Turner and Warren, 1988], five times larger than that in the case of interest. The vortex life spans, estimated by the length of KVS divided by the vortex propagation velocity, are also distinct. In the atmosphere, it is on the order of a couple of hours [Li et al., 2000], while in the studied ocean case it may last much longer, e.g., longer than five days. On the satellite images, they have distinct imagery signatures. The atmospheric KVS are usually visualized by zigzag low-level cloud patterns, but not for the OVT.

Satellite-tracked drifter trajectories and OVT signatures on ASAR images used in this study reveal that a strong current flows directly from the Pacific (Philippine Sea) to SCS passing through the south Luzon Strait in November. The average surface current velocity is around $0.7 \text{ ms}^{-1}$, and the direction is around $315^\circ$ TN. Using the methods developed in this study, the angular velocity of ocean vortex can be calculated by the radar backscatter cross section profile taken along a vortex radial transect, if the spectral constant $m_1$ is independently determined. Then, the mean flow velocity can be calculated using the OVT dynamical models.

6. Conclusions

The Argos satellite-tracked surface drifter trajectory data and ENVISAT ASAR images are used to observe the OVTs in the Luzon Strait. Two cases of vortex trains that occurred in November, local fall, are observed in the wake of Babuyan Island. The first train of three cyclonic vortices showing up on drifter trajectories is distributed in the northwest of Babuyan Islands from $20^\circ$ to $20.5^\circ$N and from...
12.0° to 121.0°E. The second train consisting of two parallel rows, five pairs of alternating cyclonic-anticyclonic vortices were observed on the ASAR images of 19 November 2006. The second train occurred on the upstream side of the first one from 19.5° to 20.0°N and from 121.0° to 122.0°E. The total length of the vortex train along the axis reaches about 250 km. All vortices propagate northwardwestward (~315° TN). The mean angular velocity is (2.07 ± 0.18)x 10^{-5} \text{s}^{-1}. The theoretical models of the ocean vortex radar images are derived from general radar imaging theories for ocean processes. These functional relations are used to extract dynamical parameters of the ocean vortex and the vortex train from ASAR imaging signatures. The results are as follows: The distances between two consecutive vortices and two rows of vortices are (22.6 ± 1.9) km and (8.2 ± 1.2) km, respectively. The maximum rotational velocity radius is determined to be 4.70 km. The vortex rotational angular velocity \omega (\text{O}(3 \times 10^{-5} \text{s}^{-1}), if the spectral constant \nu_3 is taken as 1.95 \times 10^{-3}.

[50] Dynamical analyses give reasonable estimates of OVT dynamical parameters. The current velocity of mean or undisturbed flow as \bar{U}_0 = 0.65 \text{ms}^{-1}, which coincides with the average mean flow velocity (0.70 ± 0.25) ms^{-1} measured by drifters, and the propagation velocity of vortex \bar{U}_p = 0.58 \text{ms}^{-1}. The estimated vortex shedding rate \bar{F} = \bar{U}_p/\bar{U}_0 = 2.57 \times 10^{-5} \text{s}^{-1}. The Reynolds number is \text{Re} = \bar{U}_p \ell / \nu, if \text{Re} is chosen as 1 \times 10^3 \text{cm}^{-1} \times 1 \times 10^6 \text{cm} \text{s}^{-1}. For an individual vortex, the length scale is O(5 km), and the Rossby number is O(0.4). For the OVT, the length scale is O(20 – 50 km), and the Rossby number is O(0.5), implying that vortex trains observed in the Luzon Strait are of sub-mesoscale in nature.

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