Statistical models of visual neurons

Update Presentation

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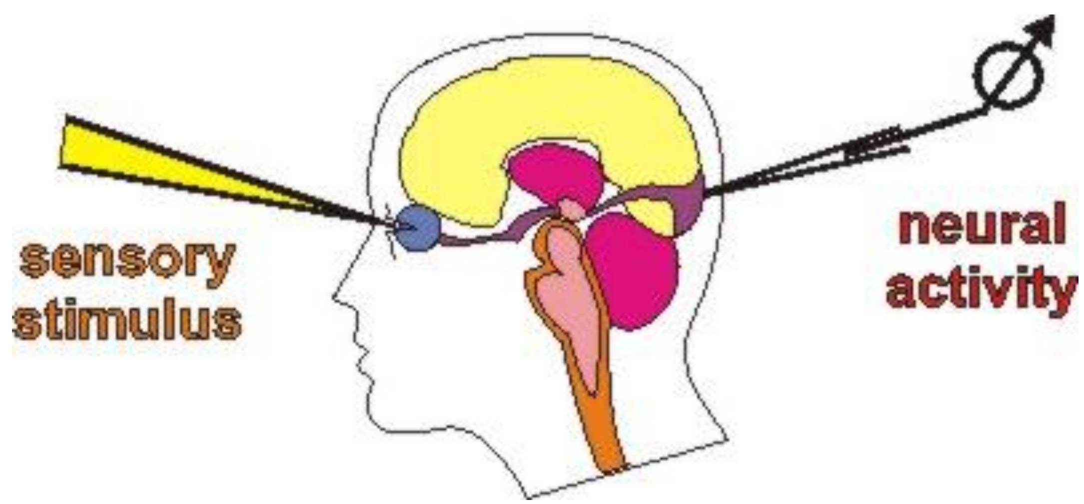
Visual system of a neuron

light



electricity

spikes

The goal of this field is to identify relationship between the visual stimuli and the resulting neural responses

http://www.pc.rhul.ac.uk/staff/j.zanker/ps1061/l2/ps1061_2.htm

Statistical modeling of neuron's response

Stimulus -> ? -> Firing rate

$\mathbf{S}(t)$ $\mathbf{r}(t)$

$$Prob(n(t)) = \frac{r(t)^{n(t)}}{n(t)!} exp(-r(t))$$

n(t) - number of spikes at moment t

What is the simplest model which describes the computation being performed by the neuron?

Previous semester models

Linear-Nonlinear-Poisson model

- Moment-based statistical models for linear filter **k** estimation:

    1. First order moment-based model - Spike Triggered Average (STA)

    2. Second order moment-based model - Spike Triggered Covariance (STC)

This semester models

Maximum Likelihood estimators:

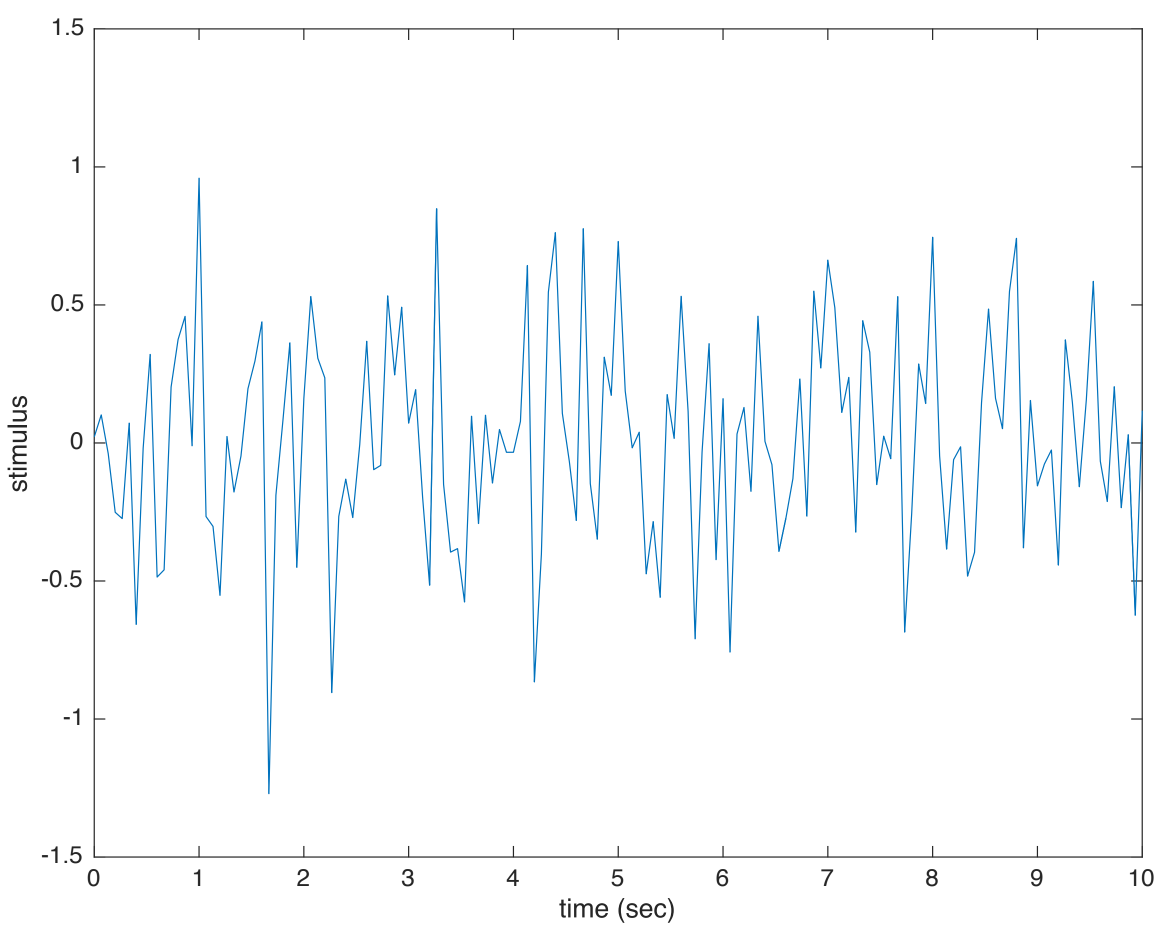
- Generalized Linear Model (GLM)
- Generalized Quadratic Model (GQM)
- Nonlinear Input Model (NIM)

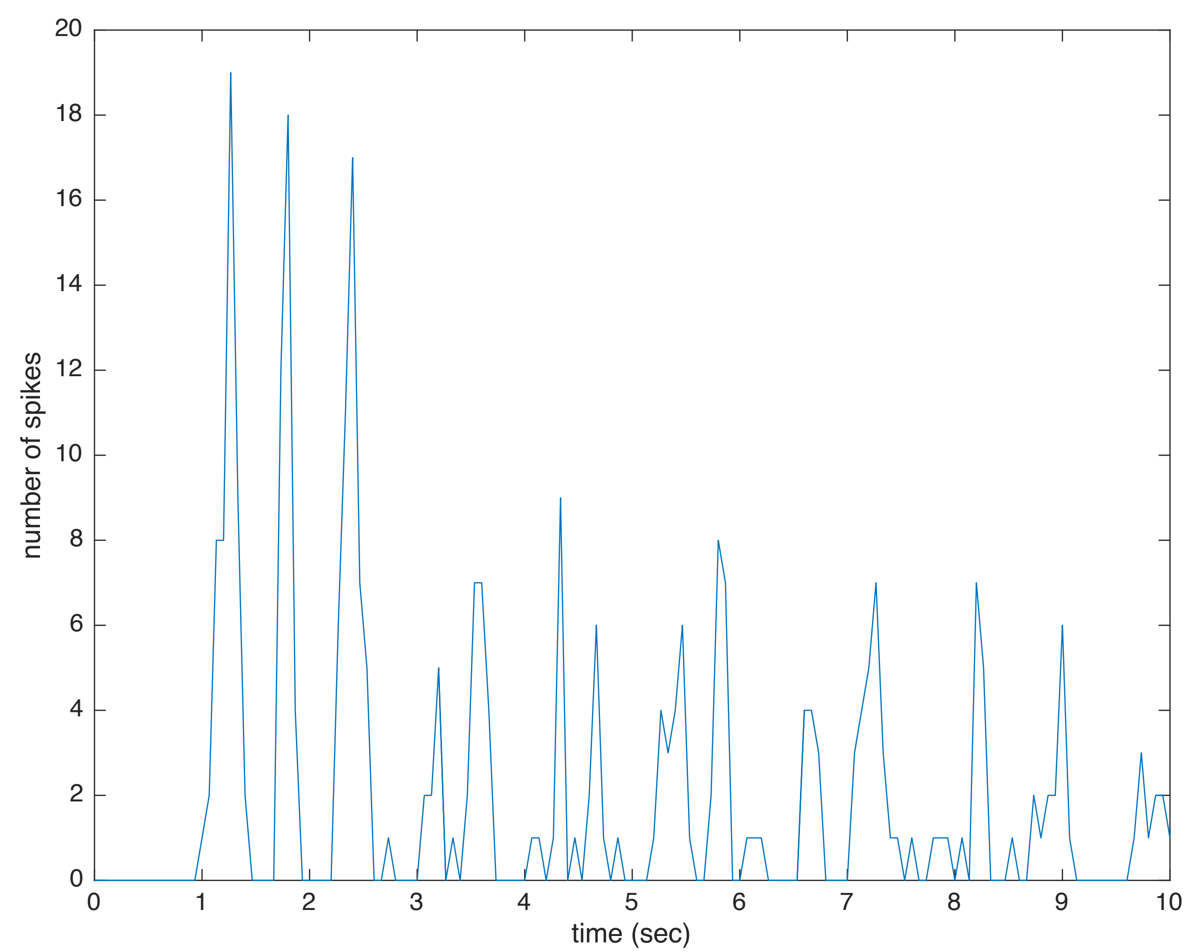
Data sets description

- Real data set - Lateral Geniculate Nucleus data (LGN)
- Synthetic data set - Retinal Ganglion Cells (RGC)

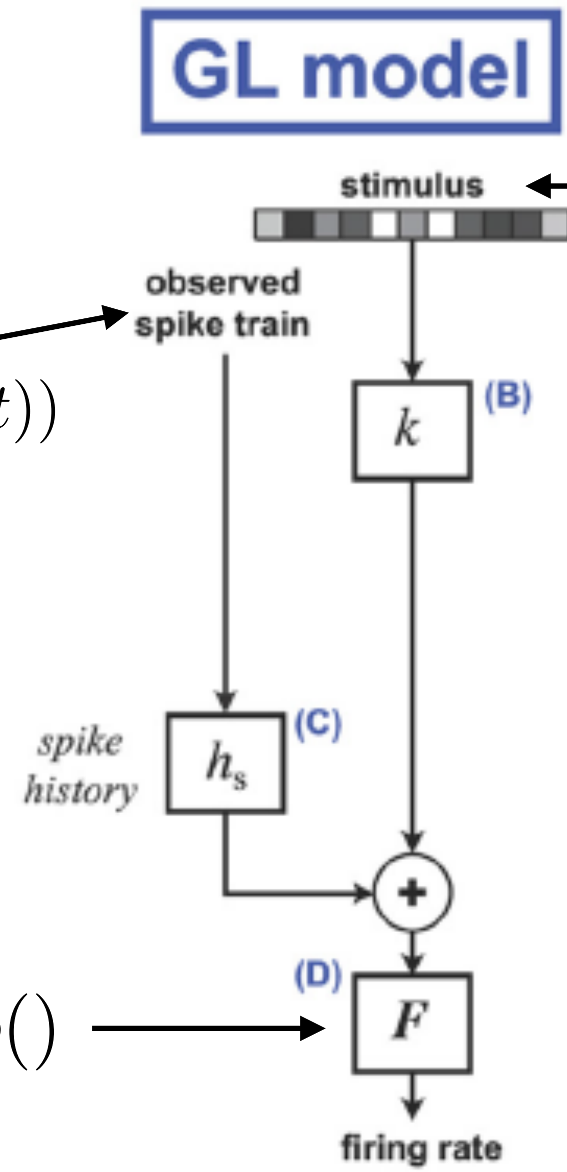
Both data sets contain :

1. Stimulus vector (**S**, which depends on time)
2. Spikes vector (**n**, which depends on time)
3. Time interval of the stimulus update (dt, time step)





Generalized Linear Model: Idea



$$\mathbf{s}(t) = (S(t-\tau), \ldots, S(t))$$

$$\mathbf{r}_{obs}(t) = (n(t-\tau), \ldots, n(t))$$

$$F() = exp()$$

Additional parameters

$$\tau = P - 1$$

P- the time window length

n - vector of spikes

S - stimulus vector

b - threshold

$$r(t) = F(\mathbf{k} \cdot \mathbf{s}(t) + \mathbf{h} \cdot \mathbf{r}_{obs}(t) + b)$$

GOAL: find optimal **k**,**h** and b

Picture reference: Butts DA, Weng C, Jin JZ, Alonso JM, Paninski L (2011) Temporal precision in the visual pathway through the interplay of excitation and stimulus-driven suppression.

Generalized Linear Model (GLM): Plan

$$r(t) = F(\mathbf{k} \cdot \mathbf{s}(t) + \mathbf{h} \cdot \mathbf{r}_{obs}(t) + b)$$

1. GLM: find LogLikelihood LL[**k**,**h**,b] -> max
2. Validate the code using simulated data
a. without h and b terms
b. with h and b terms
3. Validate part of the model(without h and b terms) comparing with STA filter for RGC data set
4. Find optimal k and h filters for LGN data set

Maximum Likelihood estimation

$$P(N|\Theta) = \prod_t \frac{(r(t))^{n(t)}}{n(t)!} exp(-r(t))$$ ← Poisson distribution

$$N = \{n(t)\} \quad \Theta = \{r(t)\}$$

$$LL(\Theta) = log(P(N|\Theta)) = \sum_t n(t) log(r(t)) - \sum_t r(t)$$

$$r(t) = F(\mathbf{k} \cdot \mathbf{s}(t) + \mathbf{h} \cdot \mathbf{r}_{obs}(t) + b) \qquad \Theta = \{\mathbf{k}, \mathbf{h}, b\} \qquad F() = exp()$$

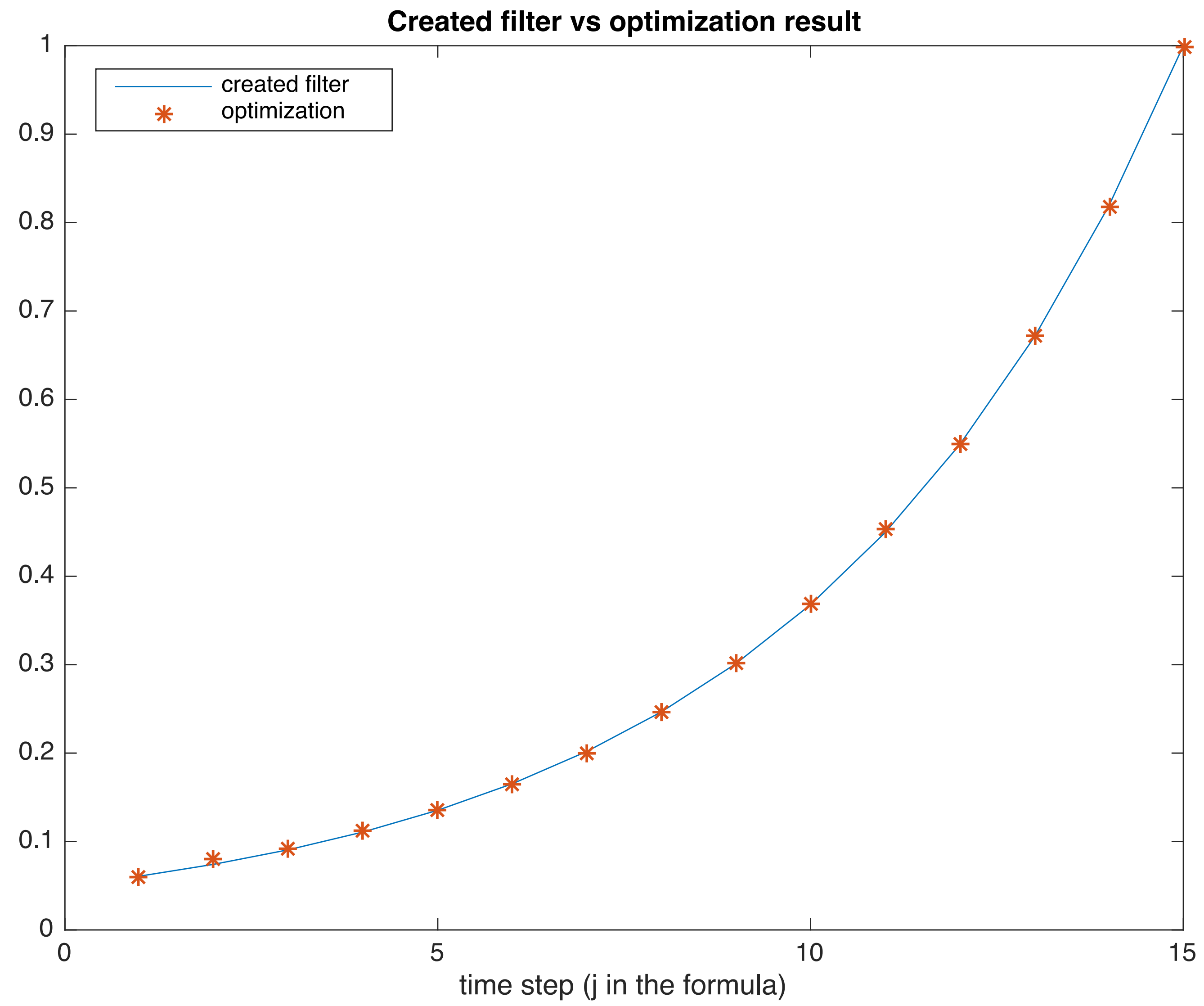
Solve log-likelihood using gradient ascent method

$$LL(\Theta) = \sum_t n(t)(\mathbf{k} \cdot \mathbf{s}(t) + \mathbf{h} \cdot \mathbf{r}_{obs}(t) + b) - \sum_t exp(\mathbf{k} \cdot \mathbf{s}(t) + \mathbf{h} \cdot \mathbf{r}_{obs}(t) + b)$$

GLM code validation:

<u>Validation:</u>

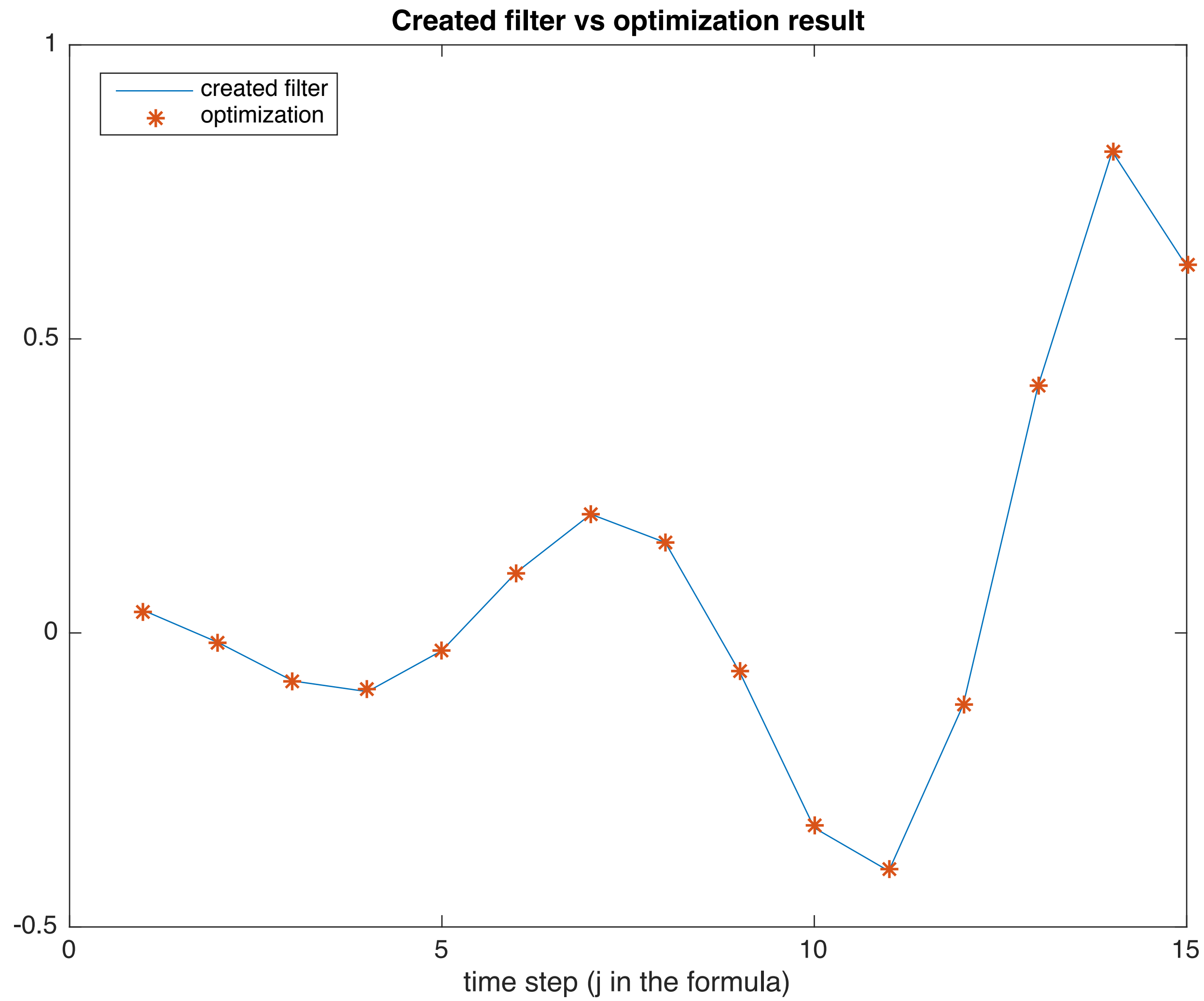
1. Generate stimulus
2. Chose a simple filter **k**
   $k_j = exp((j-15)/5)$
3. Use Poisson distribution for spike generation
4. Use generated stimulus and spikes in order to reconstruct the filter



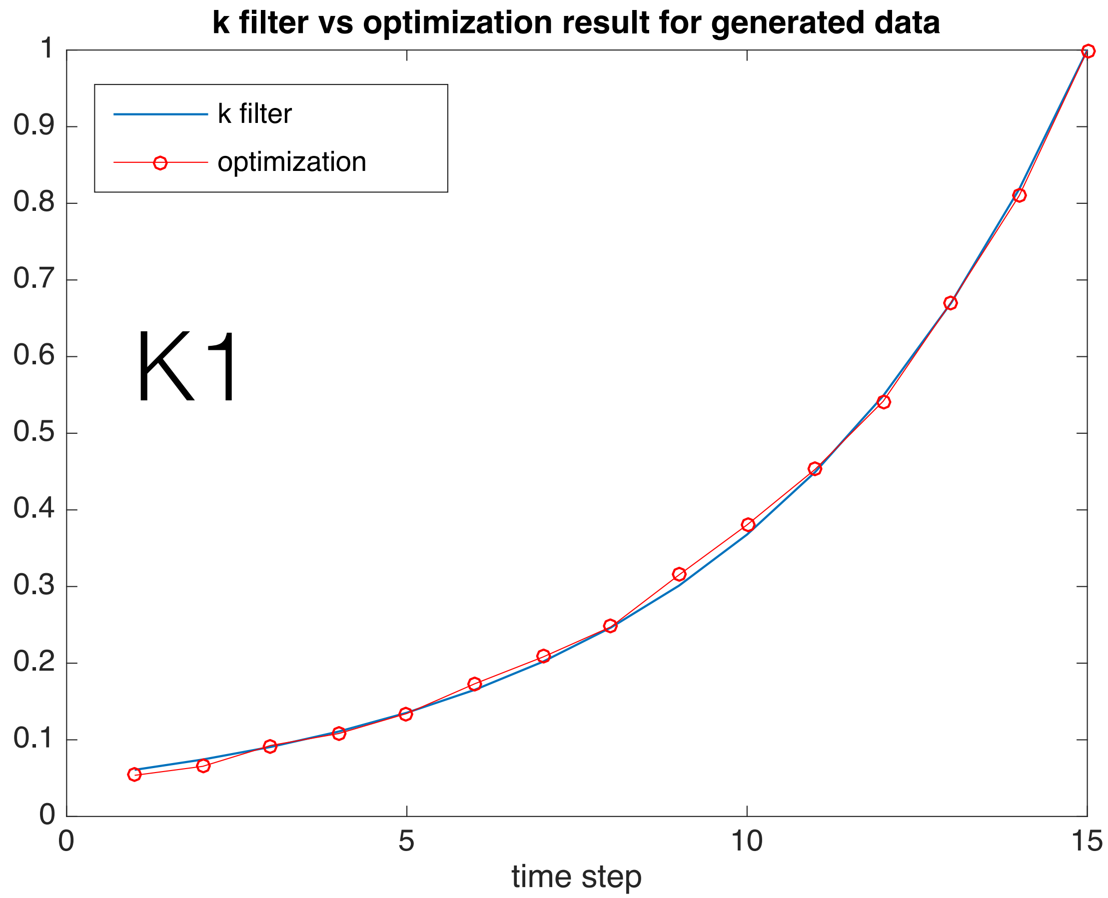
GLM code validation:

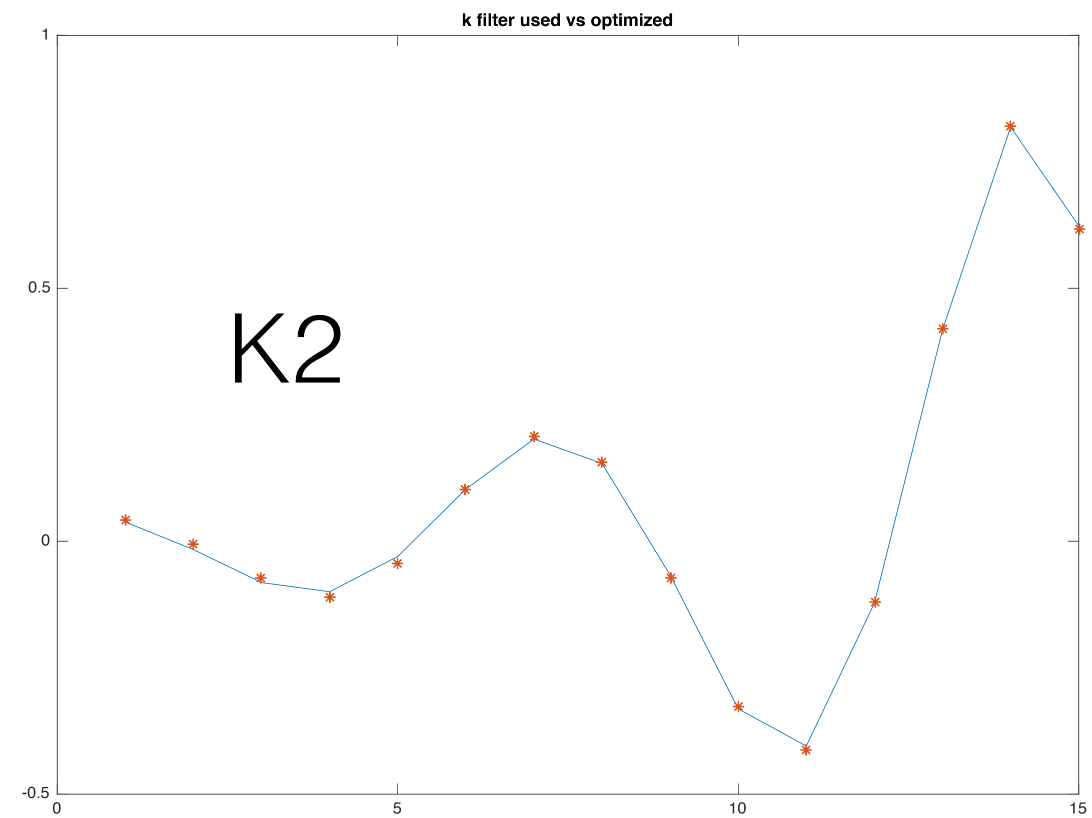
Try another test k-filter

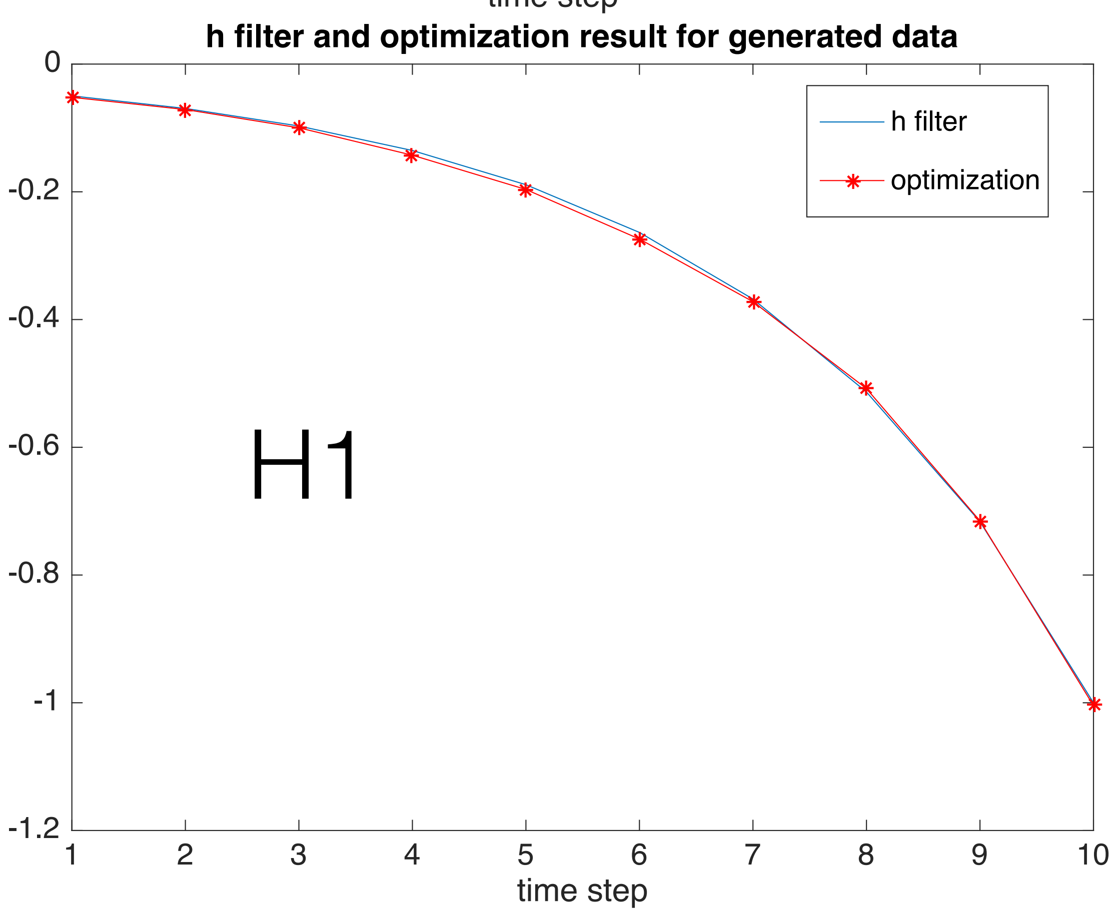
$$k_j = exp((j-15)/5) * cos(2\pi j/7)$$

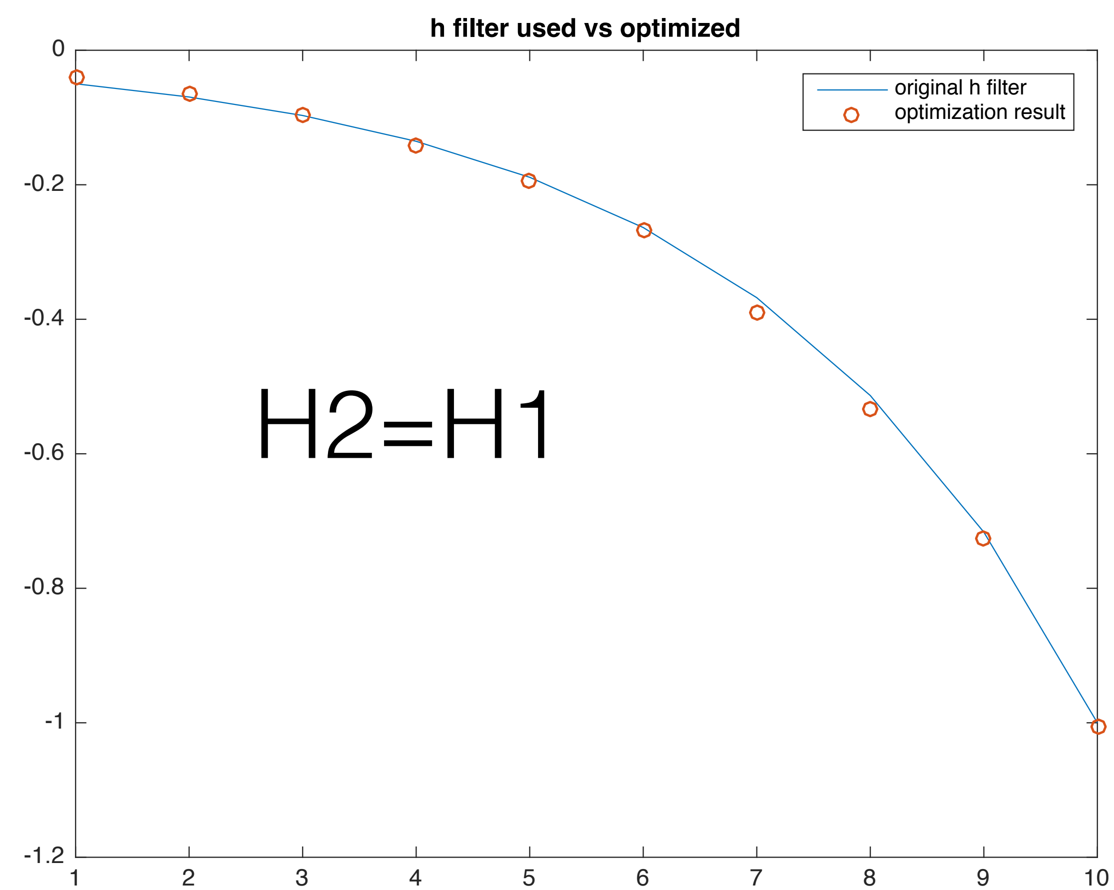


GLM code validation including history term

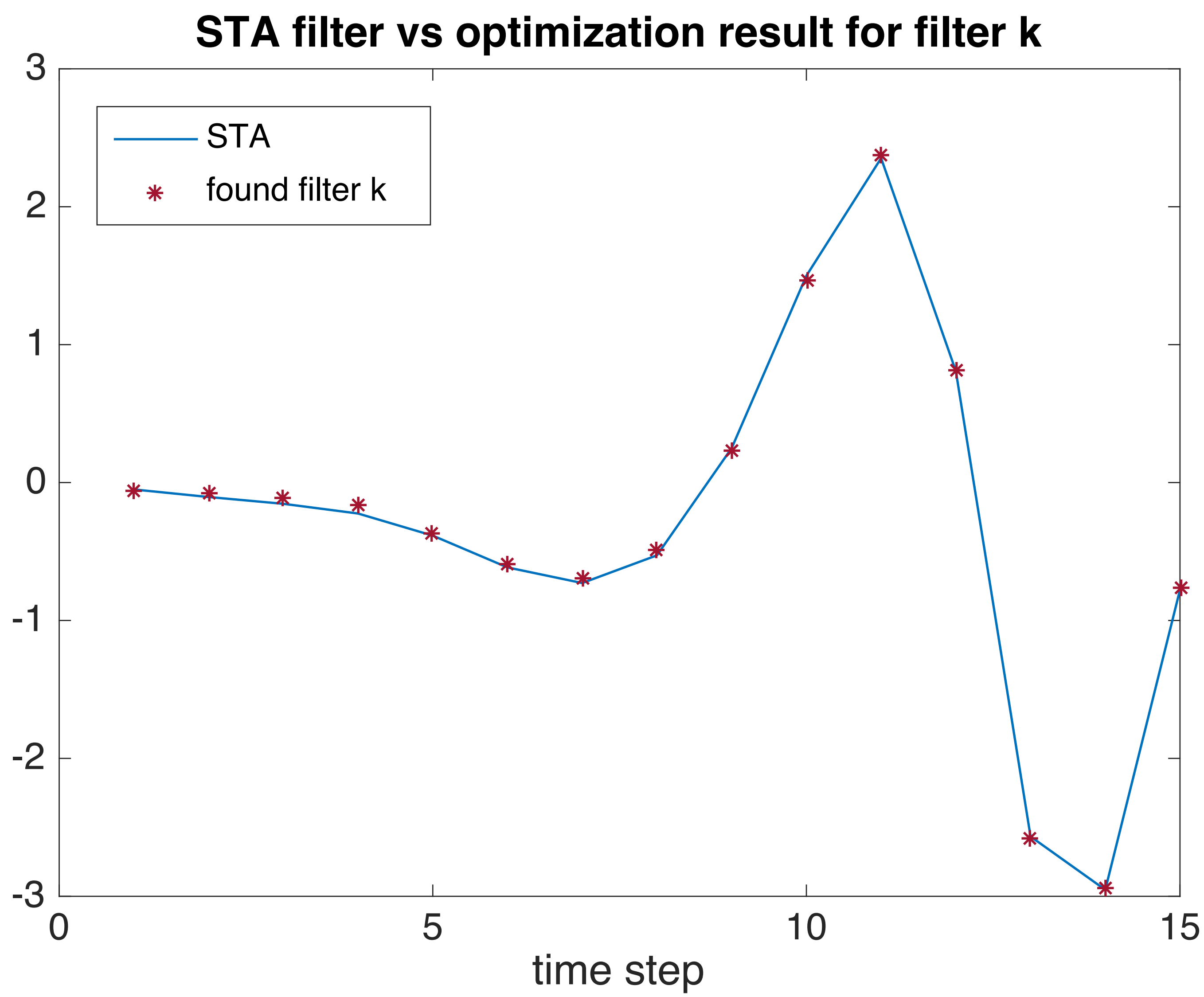




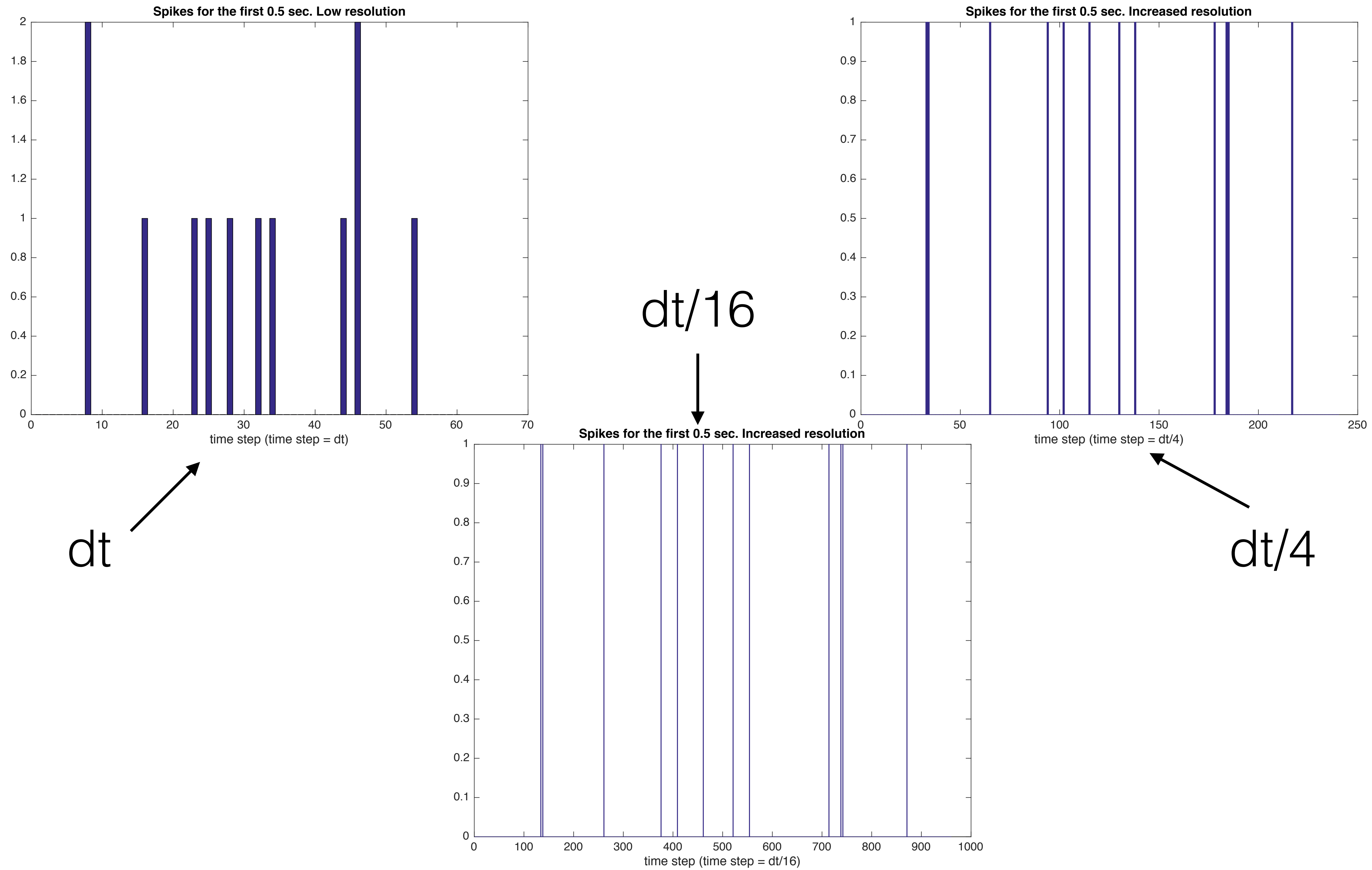




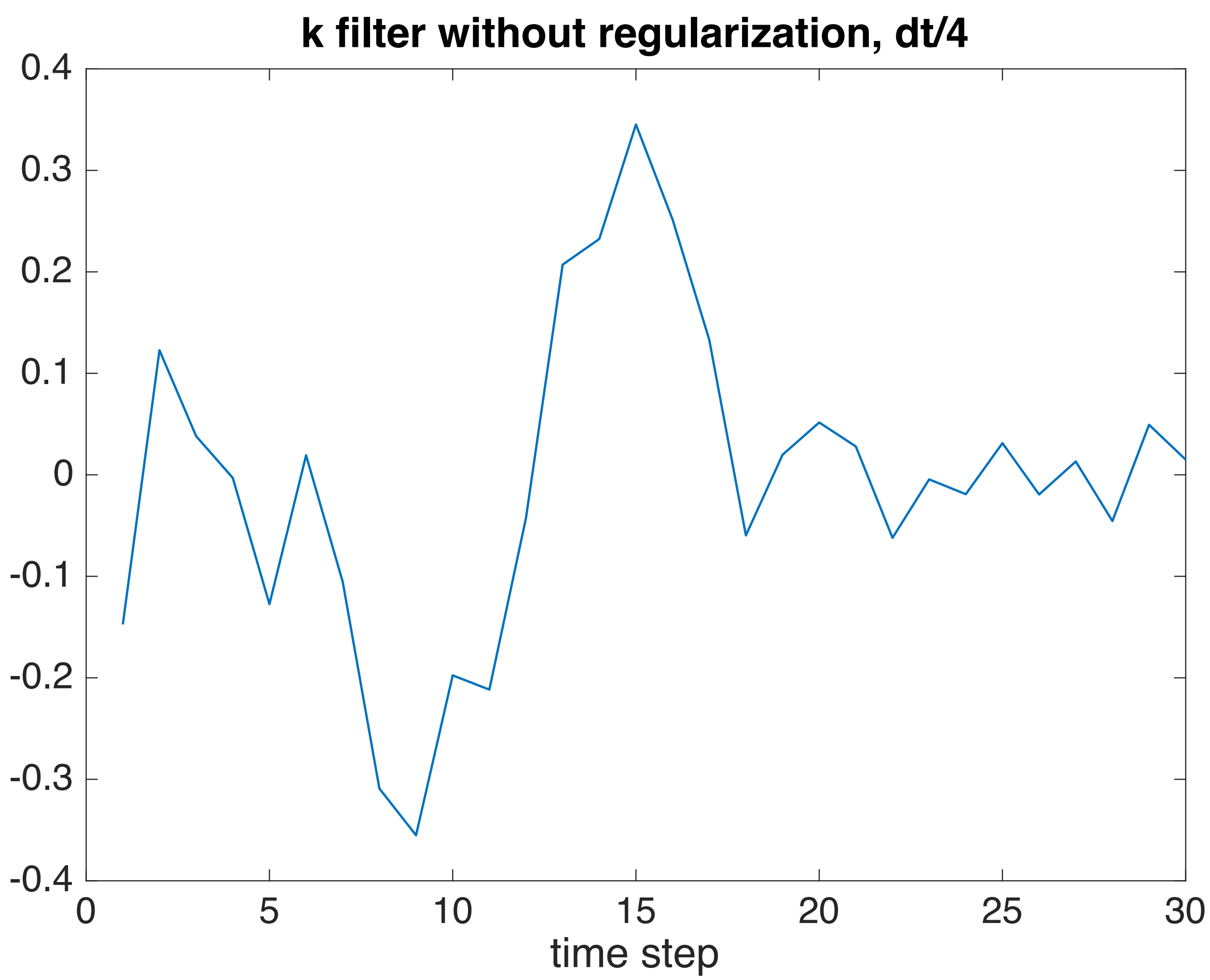
Optimal linear filter matches STA for RGC data:



Need higher time resolution to catch history term



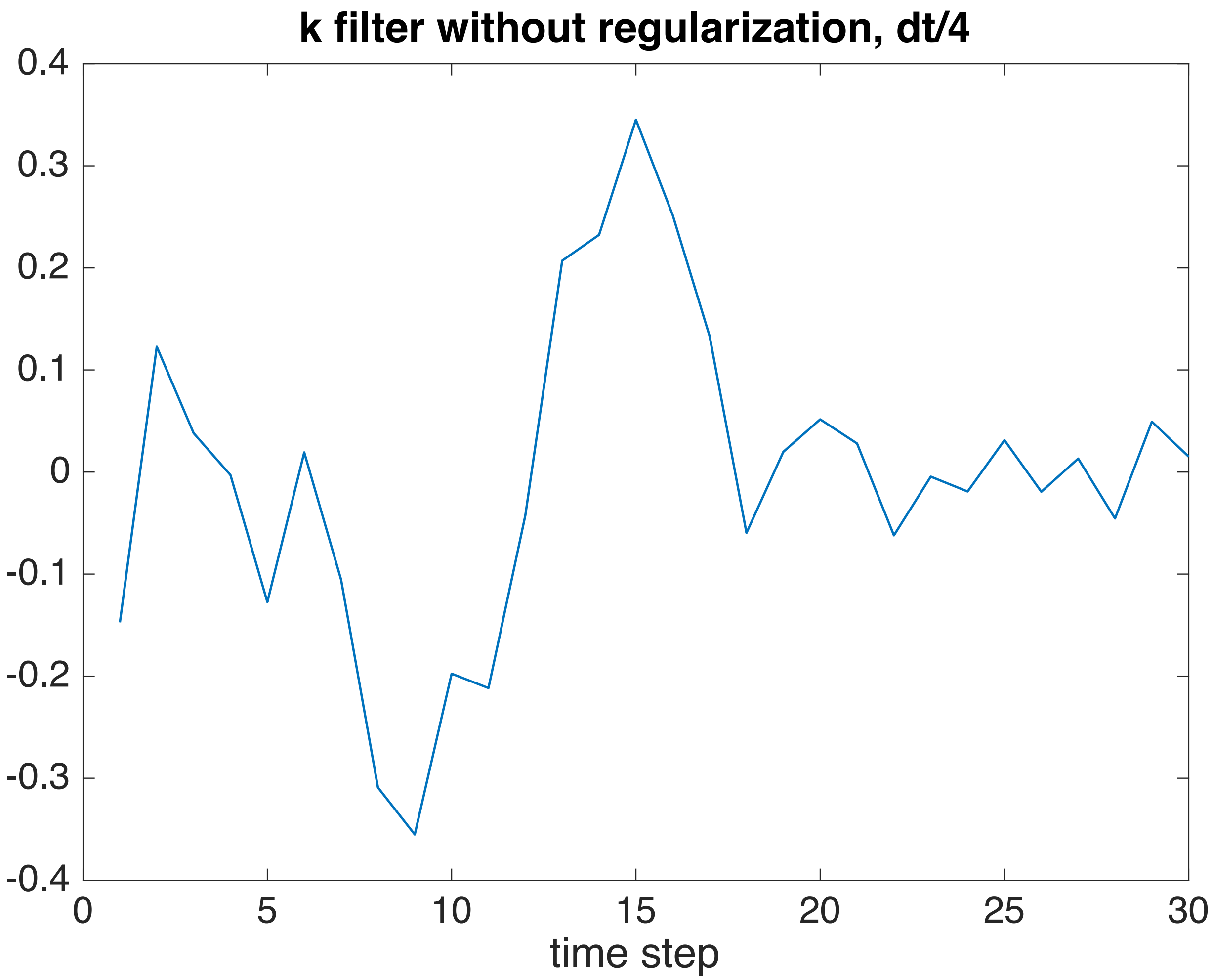
Regularization is needed at high resolutions



Solve regularized log-likelihood using gradient ascent method

$$RLL(\Theta) = \sum_t n(t)(\mathbf{k}\cdot\mathbf{s}(t) + \mathbf{h}\cdot\mathbf{r}_{obs}(t) + b) - \sum_t exp(\mathbf{k}\cdot\mathbf{s}(t) + \mathbf{h}\cdot\mathbf{r}_{obs}(t) + b) - \lambda\sum_i (k_i - k_{i-1})^2$$

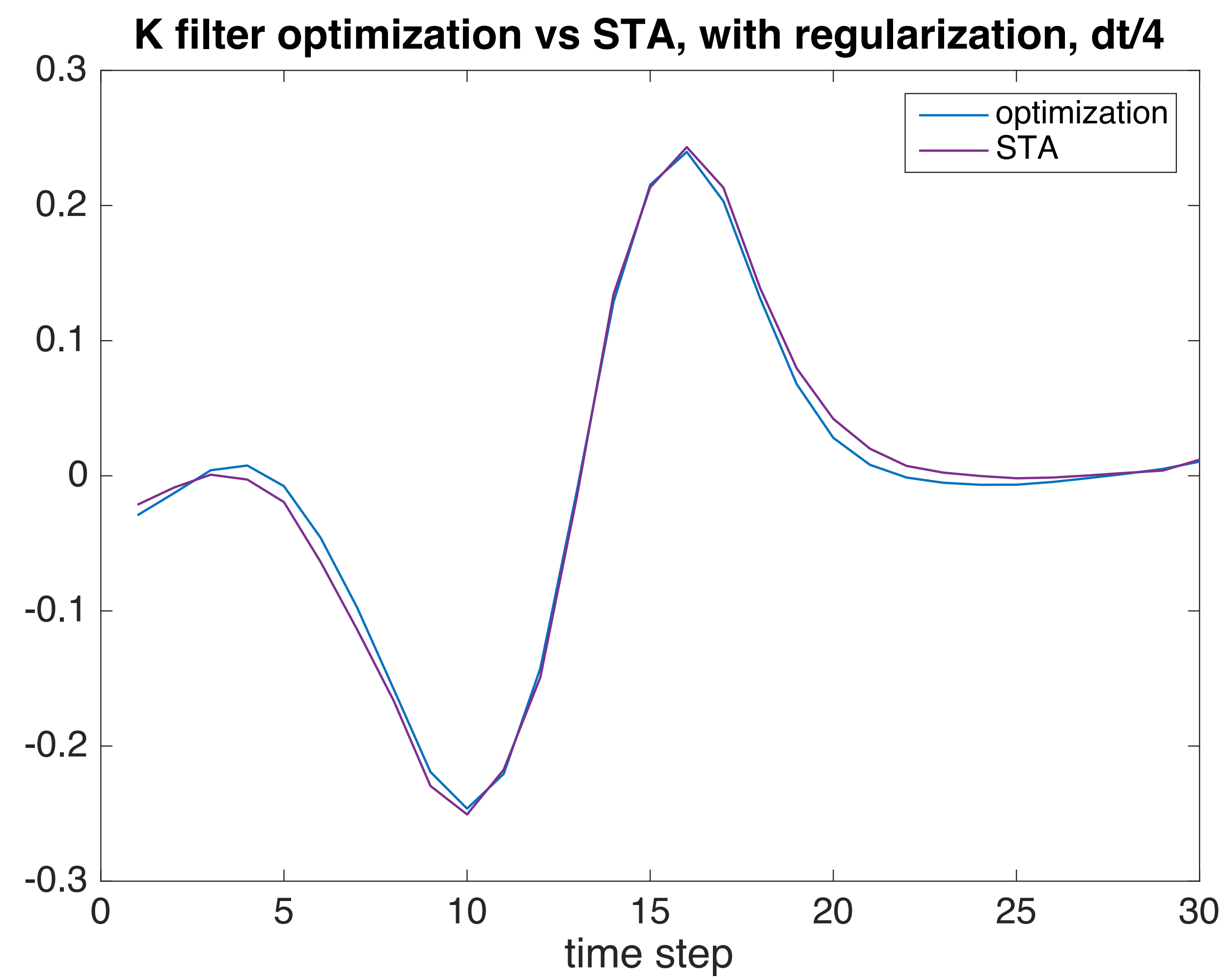
GLM implementation: increase the resolution.



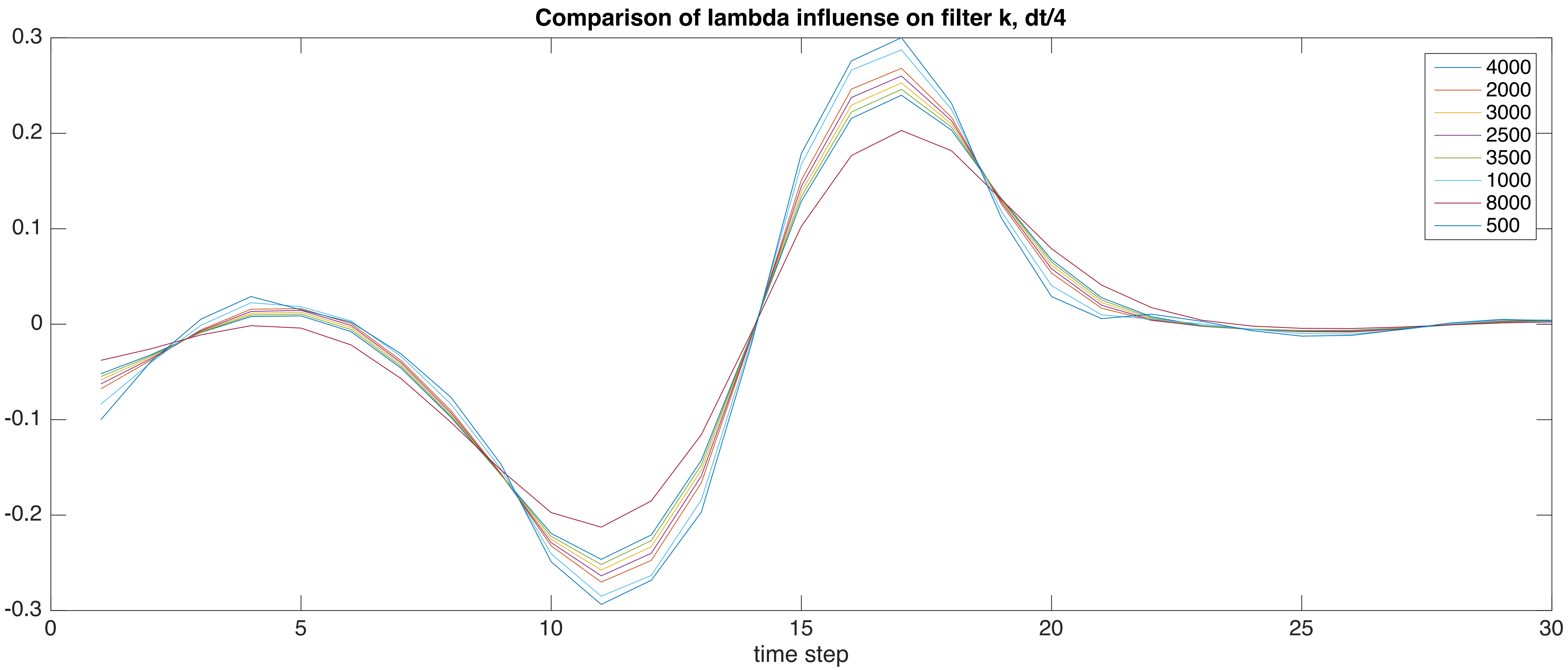
Resolution is dt/4, no history term.

LL ~ 10^4
Regularization term ~ 10^2

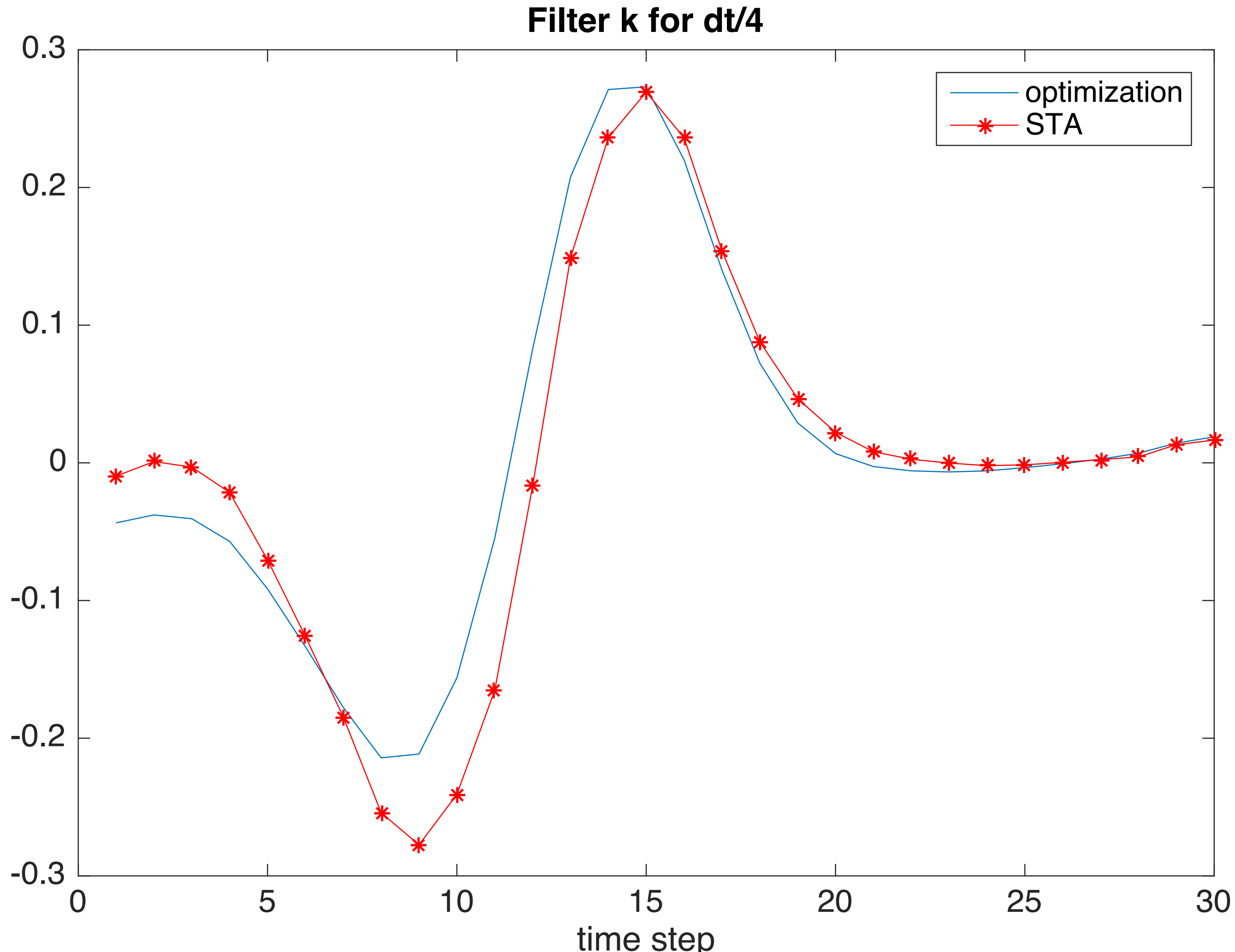
max difference between STA and optimization result is about 1%

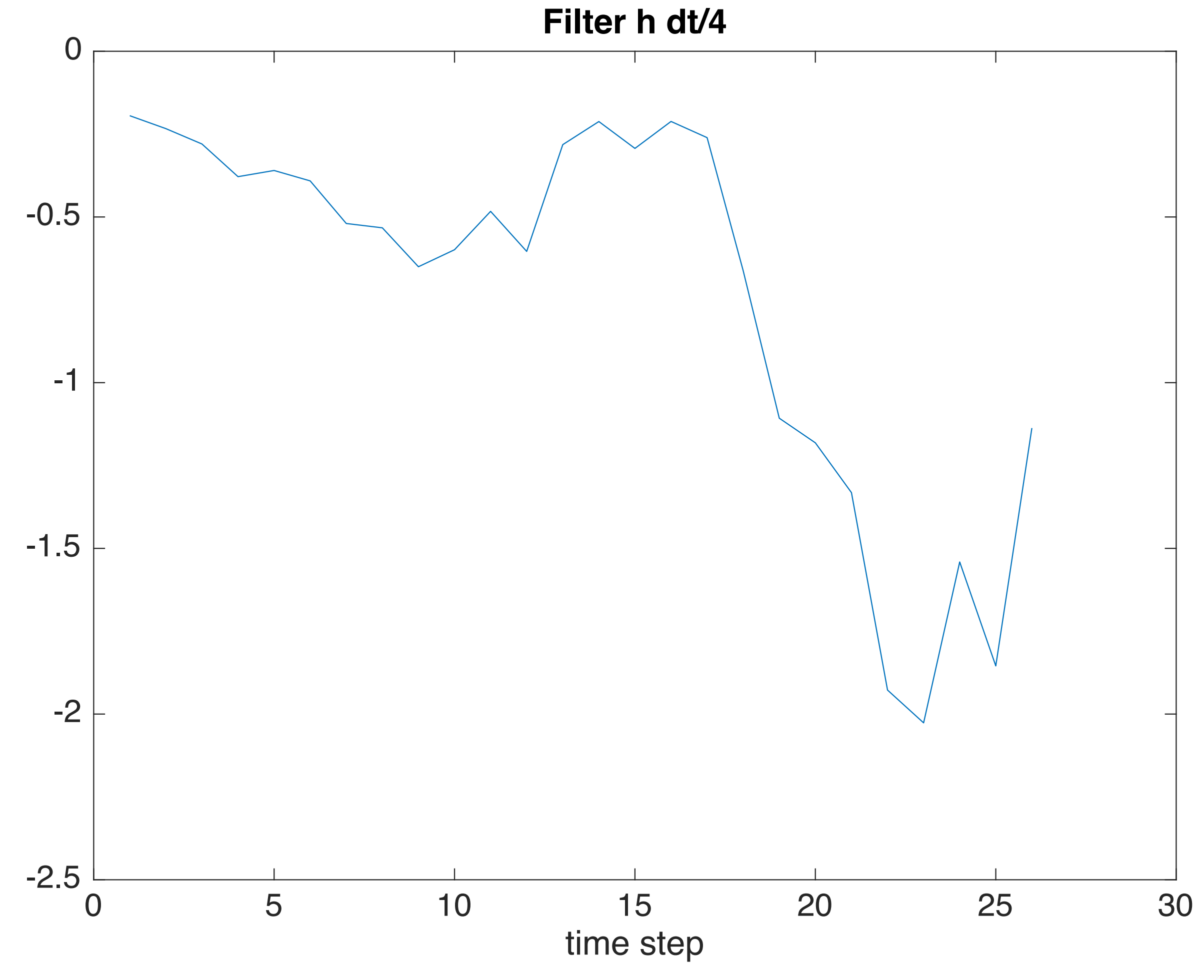


GLM implementation: regularization parameter choice



GLM with k,h and b parameters



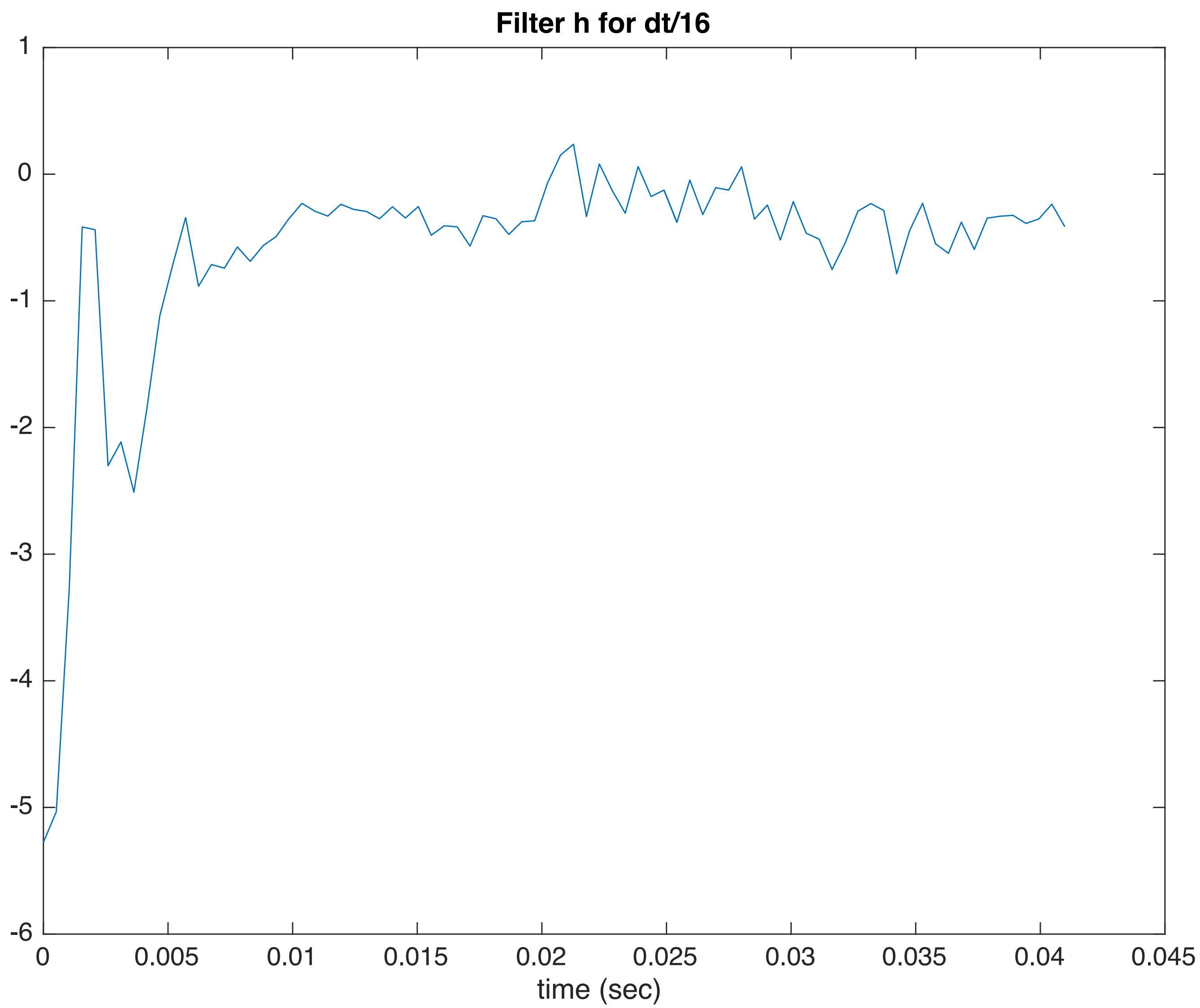


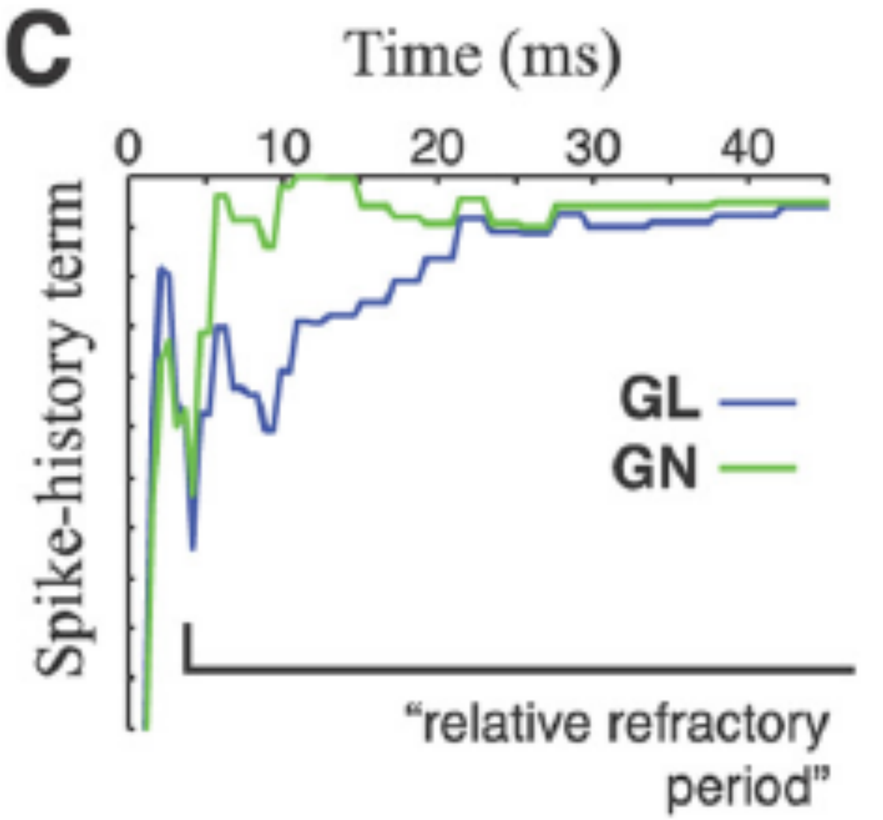
OK

Not OK

Resolution
dt/4

GLM with k,h and b parameters





neuron's history detected for resolution dt/16

Picture reference: Butts DA, Weng C, Jin JZ, Alonso JM, Paninski L (2011) Temporal precision in the visual pathway through the interplay of excitation and stimulus-driven suppression.

Summary

- GLM algorithm was implemented and validated on generated data
- The history term was extracted using regularization of log-likelihood
- Optimization of up to 240 parameters simultaneously was implemented

Updated project schedule

October - ~~mid November~~ November

- ✓ Implement STA and STC models
- ✓ Test models on synthetic data set and validate models on real data set

~~November - December~~ December - mid February

- ✓ Implement Generalized Linear Model (GLM)
- ✓ Test model on synthetic data set and validate model on LGN data set

~~January - March~~ mid February - mid April

- Implement Generalized Quadratic Model (GQM) and Nonlinear Input Model (NIM)
- Test models on synthetic data set and validate models on LGN data set

~~April - May~~ Mid April - May

- Collect results and prepare final report

References

1. McFarland JM, Cui Y, Butts DA (2013) Inferring nonlinear neuronal computation based on physiologically plausible inputs. PLoS Computational Biology 9(7): e1003142.

2. Butts DA, Weng C, Jin JZ, Alonso JM, Paninski L (2011) Temporal precision in the visual pathway through the interplay of excitation and stimulus-driven suppression. J. Neurosci. 31: 11313-27.

3. Simoncelli EP, Pillow J, Paninski L, Schwartz O (2004) Characterization of neural responses with stochastic stimuli. In: The cognitive neurosciences (Gazzaniga M, ed), pp 327–338. Cambridge, MA: MIT.

4. Paninski, L., Pillow, J., and Lewi, J. (2006). Statistical models for neural encoding, decoding, and optimal stimulus design.

5. Shlens, J. (2008). Notes on Generalized Linear Models of Neurons.

Appendix: Gradient in GLM with k,h and b (including regularization term)

for i = 2 to M-1 (except the1st and the last elements), where M is the number of stimuli = stimulus_length - P +1

$$\frac{dLL}{dk_i} = \sum_t n_t s_{it} - \sum_t exp(\mathbf{k} \cdot \mathbf{s}_t + \mathbf{h} \cdot \mathbf{r}_{obs_t} + b) * s_{it} - 2 * \lambda(k_i - k_{i-1}) + 2 * \lambda(k_{i+1} - k_i)$$

$$\frac{dLL}{dk_1} = \sum_t n_t s_{1t} - \sum_t exp(\mathbf{k} \cdot \mathbf{s}_t + \mathbf{h} \cdot \mathbf{r}_{obs_t} + b) * s_{1t} + 2 * \lambda(k_2 - k_1)$$

$$\frac{dLL}{dk_M} = \sum_t n_t s_{Mt} - \sum_t exp(\mathbf{k} \cdot \mathbf{s}_t + \mathbf{h} \cdot \mathbf{r}_{obs_t} + b) * s_{Mt} - 2 * \lambda(k_M - k_{M-1})$$

$$\frac{dLL}{dh_i} = \sum_t n_t r_{obs_{it}} - \sum_t exp(\mathbf{k} \cdot \mathbf{s}_t + \mathbf{h} \cdot \mathbf{r}_{obs_t} + b) * r_{obs_{it}}$$

$$\frac{dLL}{db} = \sum_t n_t - \sum_t exp(\mathbf{k} \cdot \mathbf{s}_t + \mathbf{h} \cdot \mathbf{r}_{obs_t} + b)$$